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COMPENSATION OF CONTROL SYSTEMS UTILIZING ACCELERATION FEEDBACK

JOSEPH D. BRONZINO

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COMPENSATION OF CONTROL SYSTEMS
UTILIZING
ACCELERATION FEEDBACK

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by

Joseph D. Bronzino

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Joseph D. Bronzino

Submitted in partial fulfillment of
the requirements for the degree of

MASTER OF SCIENCE
IN
ELECTRICAL ENGINEERING

United States Naval Postgraduate School
Monterey, California

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ABSTRACT

In the field of feedback compensation, there are a variety of techniques available to achieve desired results. However, a new design technique is introduced by which the process of compensation becomes a simple process rather than one plagued by trial and error.

Once this technique is discussed, acceleration feedback compensation is used exclusively, and a general rule as well as a nomogram is developed for the compensation of second order systems. Higher order systems are also considered and the most interesting of cases noted.

Among the techniques used by the author to establish the various conclusions reached, analogue computer simulation and digital computer solution of characteristic equations was employed and is discussed in the Appendices.

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INTRODUCTION

If a system is to be made stable or must be altered to meet the specifications set by a certain application, it is said to be in need of compensation. At present, there are a variety of methods available that enable this compensation to be achieved. However, there is a great deal of trial and error in many of these techniques; and as a result they leave quite a $\overset{b}{v}$it to be desired when it comes to energy and time spent in obtaining the desired result. Therefore, in this thesis we would like to exhaust the possibilities available using feedback compensation techniques in the hope of establishing a set of rules that can be followed by the design engineer in order to compensate various type systems.

The method of attack to be employed is one in which the entire system is reduced to an equivalent single block in cascade with some unknown compensator. Then the unknown compensator can be selected rather easily to yield a system that responds as desired. Chapter One deals primarily with the concept of this method, and discusses its application to multi-order systems.

There are many practical methods of feedback compensation, two of which are first derivative feedback compensation, and second order or acceleration feedback compensation, and these are among the most important possibilities. This paper deals primarily with the application of acceleration feedback to compensate multi-order systems. Chapter Two and Four illustrate the effect of this type of compensation on second order and multi-order systems respectively.

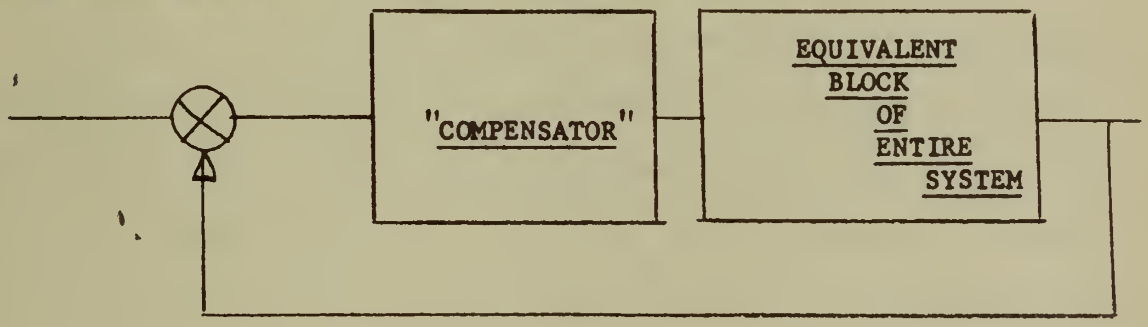
For the second order case it was discovered that a set of les can be derived and a graph developed from which the compensation problem is easily solved once the initial and desired set of conditions are established. Chapter Three presents this "Nomogram" along with several examples to illustrate its use. It should be noted however that its use need not be confined just to second order systems, but may prove extremely helpful in compensating third systems which have a real root that is far out on the negative real axis.

It is hoped that by this presentation the areas of interest pertaining to acceleration feedback may be completely brought to light. Once this is done, this type of compensation can either be readily used or rapidly discarded as a possible compensation scheme.

CHAPTER ONE

THE CASCADE COMPENSATOR

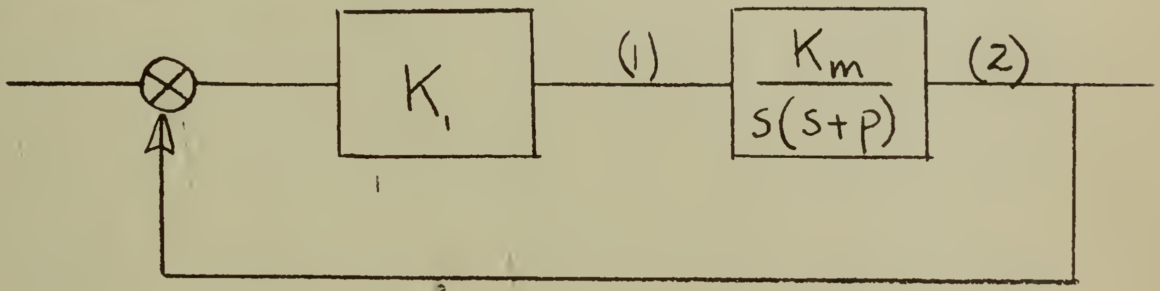
A new technique is to be presented in this chapter by which the entire system can be reduced to an equivalent block in "Cascade with a Compensator." This can be accomplished by opening a loop of the system at some point to which it is desired to feedback a signal (or where it is convenient to pick off a signal and feed it back), reduce all the remaining blocks to an equivalent block and then close the loop. In so doing, the desired condition of the entire system represented by its equivaalent block in cascade with a compensator will be obtained. This is strictly general in nature, and can be applied to any system. This technique now leaves the compensator problem in the following form:



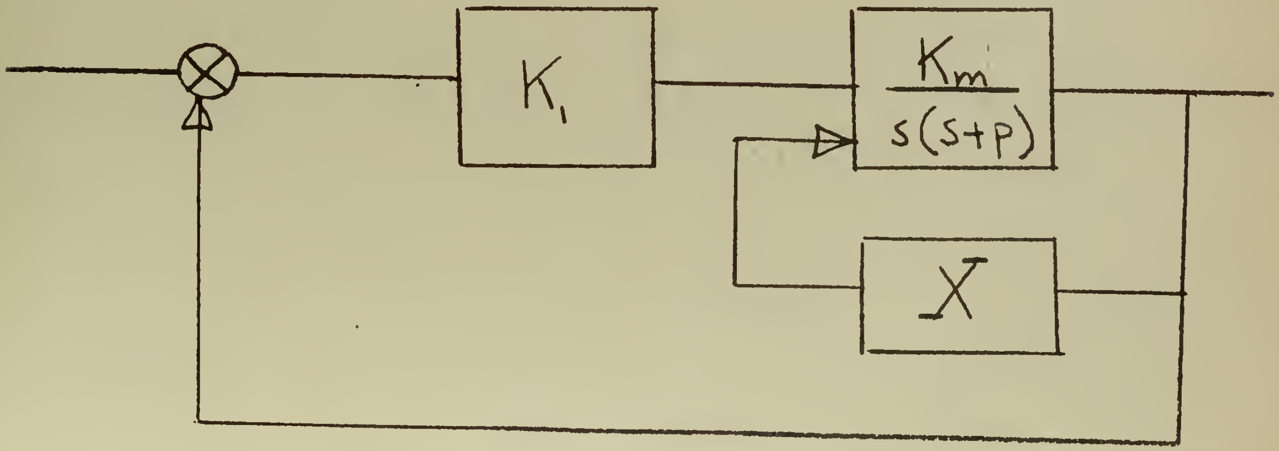
Then the characteristic equation of the resultant system can be obtained very easily for each compensator that we may desire to use by simply performing block diagram addition. Thus, knowing what the specifications are, a compensator can be selected of such a form that will yield a resultant characteristic equation having roots that satisfy the initial specifications. It is a technique that apparently eliminates a lot of time, since many compensators can be discarded rather swiftly. It also cuts down on the labor involved, since the reduction need be performed only

once and then the various compensators may be tested without again having to reduce the system, unless it is desired to select another place for the compensator.

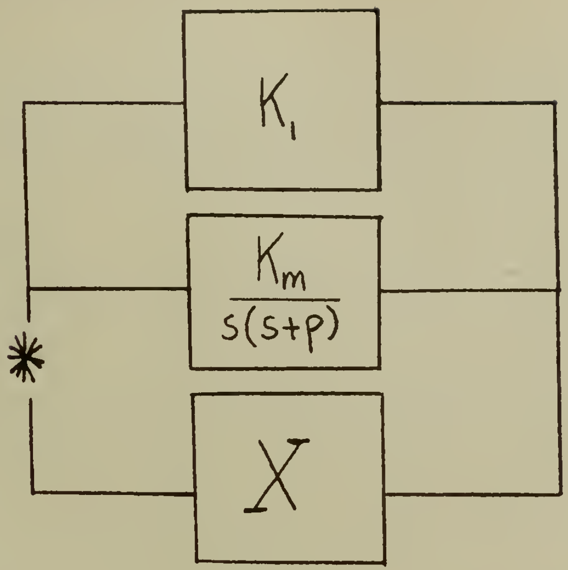
At this point, it would be best to illustrate the above discussion by considering several simple systems. First consider a second order system having unity feedback.



Now suppose that for some reason it is desired to use compensation on this system to improve its performance. Say, for example that its damping ratio (ζ) is much too low and must be improved if it is to be used in a certain application. Therefore, selecting point (2) and point (1) as convenient pick off and feed back points, it is desired to find out what type of compensator should be used between these two points to obtain the desired results.



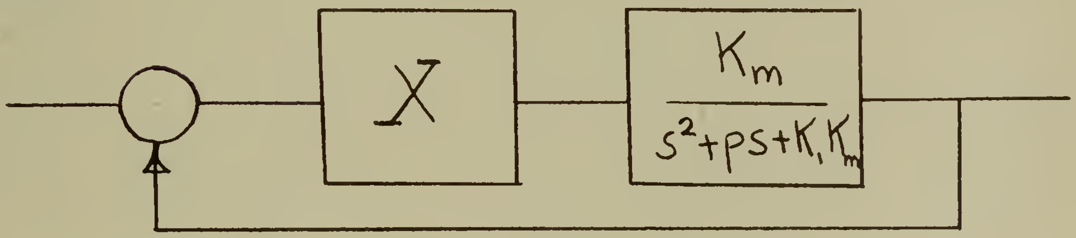
Thus by opening the loop at point *



Combining K_1 & $\frac{K_m}{s(s+p)}$

$$G_{\text{EQUIVALENT}} = \frac{\frac{K_m}{s(s+p)}}{1+\frac{K_1 K_m}{s(s+p)}} = \frac{K_m}{s^2+ps+K_1K_m}$$

Therefore

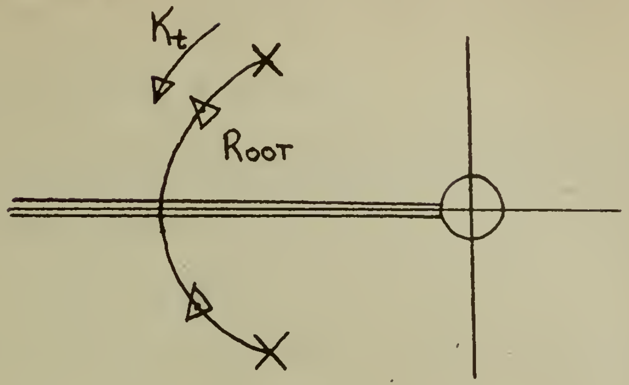


Thus now a way of determining the characteristic equation of the entire system for any compensator placed between points (1) and (2) of the system is available. For example selecting X to be tachometer feedback, that is, $X = K_t s$

Let's see what the characteristic equation is:

$$1+KGH = 1 + \frac{K_t K_m s}{s^2+ps+K_1K_m} = 0$$

This will give a root locus of the following form:



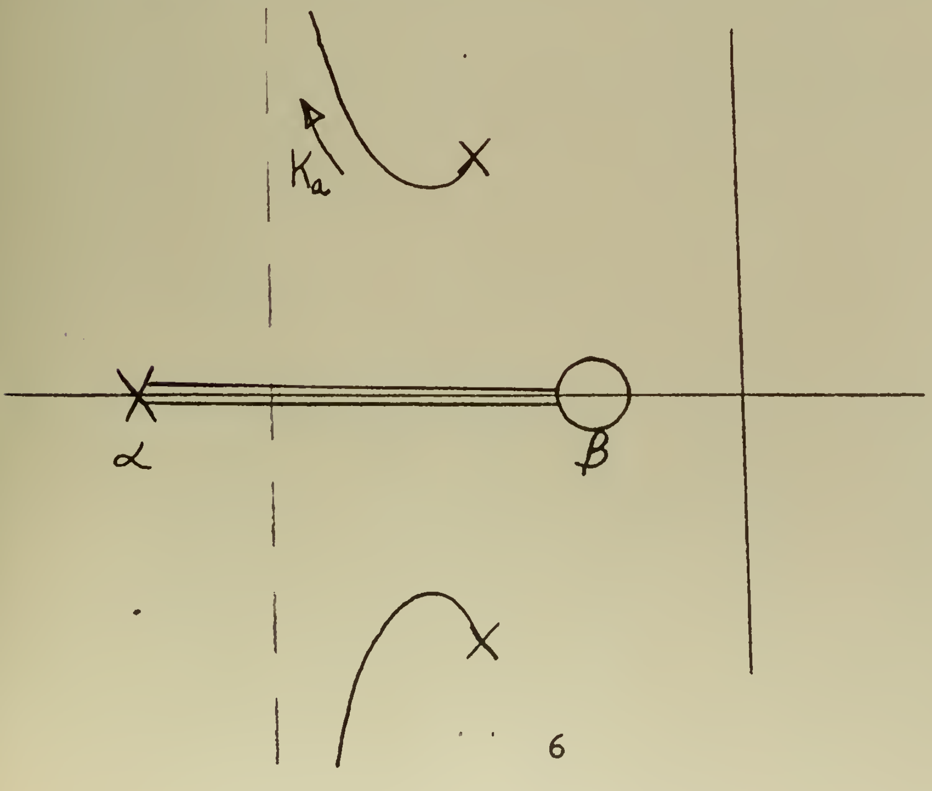
and the roots that are desired may be selected by variation of K_t. In any event the effects of adding this type of compensator to our system can readily be seen.

If we want to make $X = \frac{K_a(s+\beta)}{(s+\alpha)}$

the characteristic equation would be

$$1 + KGH = 1 + \frac{K_a K_m (s+\beta)}{(s+\alpha)(s^2 + \rho s + K_1 K_m)} = 0$$

which results in the following:

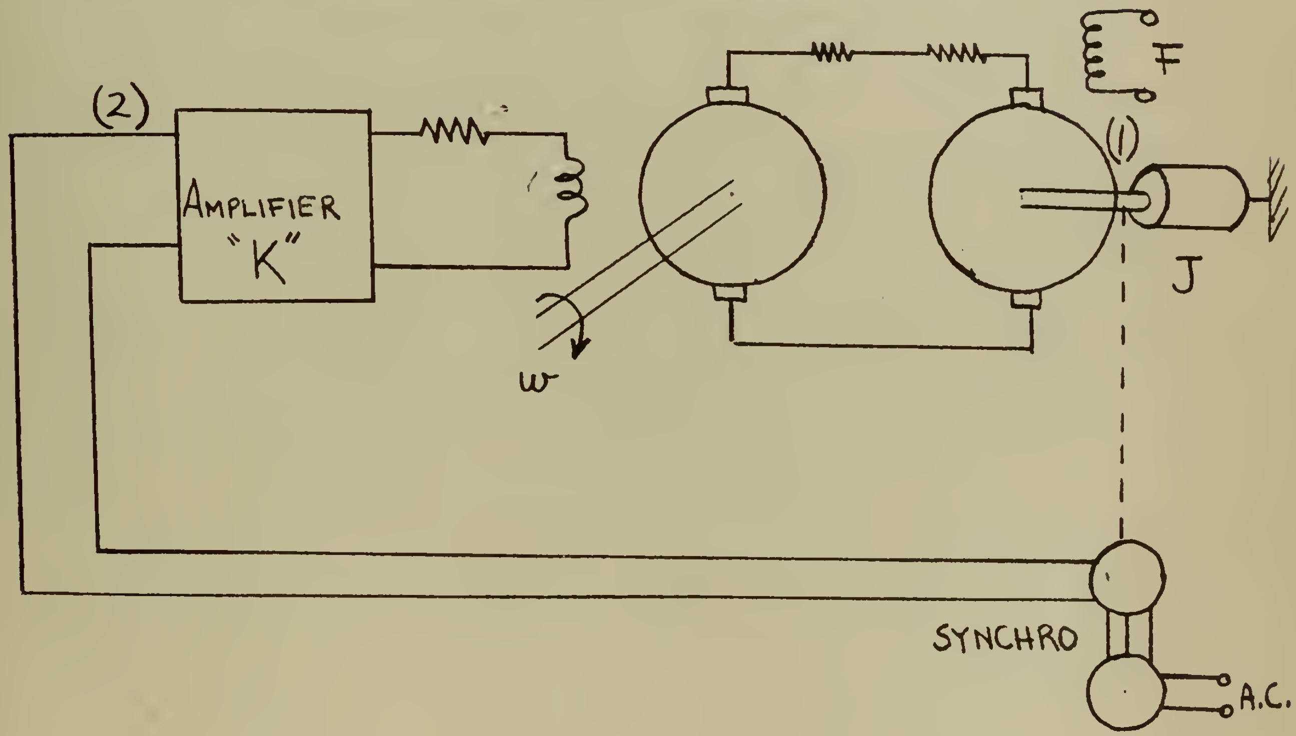


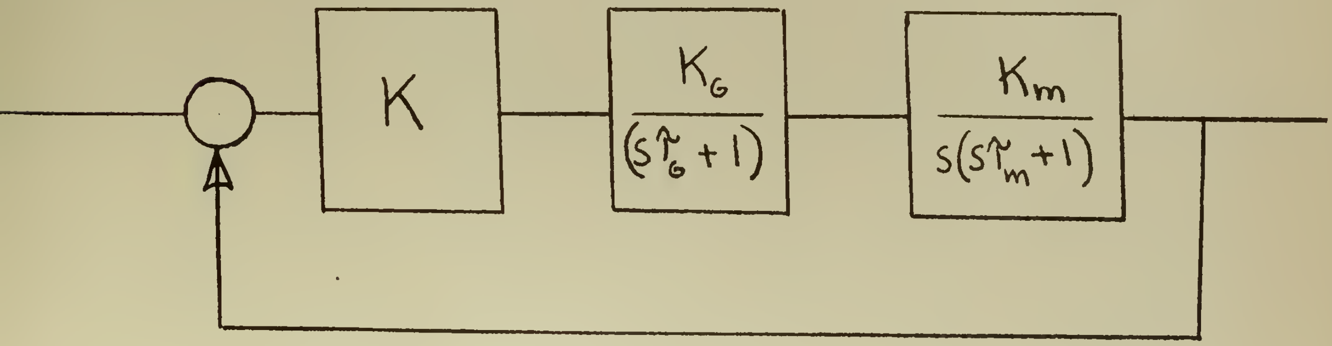
so that perhaps only for certain values of feedback gain could the system's characteristic be improved.

However, the idea has been established that a study of the effect of numerous compensating devices can be accomplished easily once the entire system has been reduced to this form.

Let's then extend this method of attack to more complicated problems so that the ease in handling even the more complex systems may be pointed out.

Suppose that the system to begin with is as shown below:





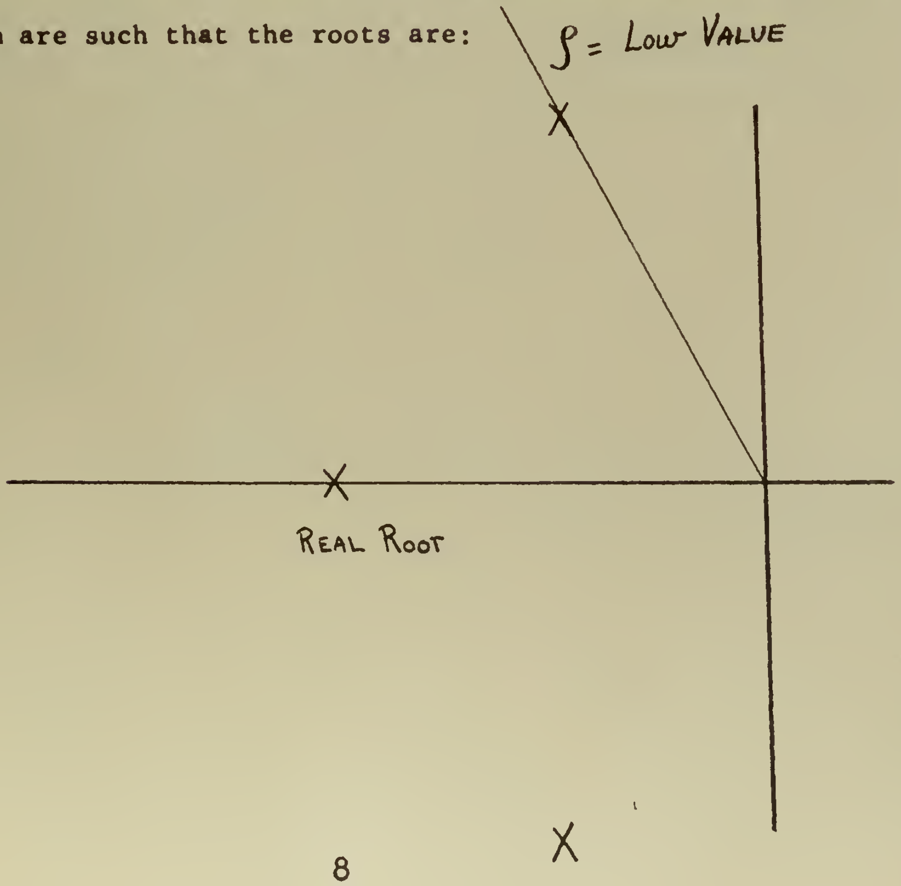
so that:
$$\frac{\theta_c}{\theta_R} = \frac{\dfrac{K_G K_m K}{s(s\tau_G+1)(s\tau_m+1)}}{1+\dfrac{K_G K_m K}{s(s\tau_G+1)(s\tau_m+1)}}$$

$$= \frac{\dfrac{K_{TOTAL}}{s(s+\alpha)(s+\beta)}}{1+\dfrac{K_{TOTAL}}{s(s+\alpha)(s+\beta)}}$$

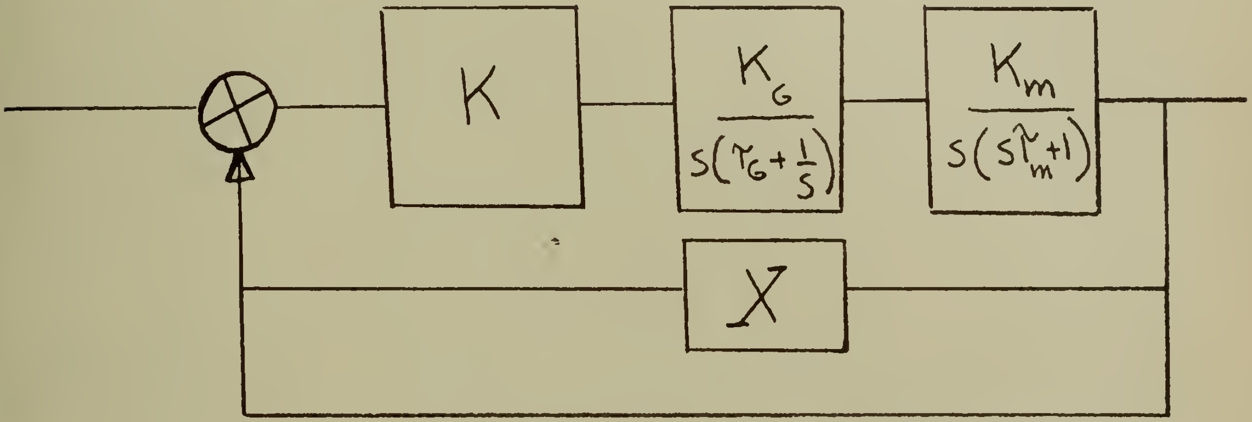
$$= \frac{K_{TOTAL}}{s^3+s^2(\beta+\alpha)+\beta\alpha s+K_{TOTAL}}$$

$$K_{TOTAL} = \frac{K_G K_m K}{\tau_G \tau_m} \qquad \alpha = \frac{1}{\tau_G} \qquad \beta = \frac{1}{\tau_m}$$

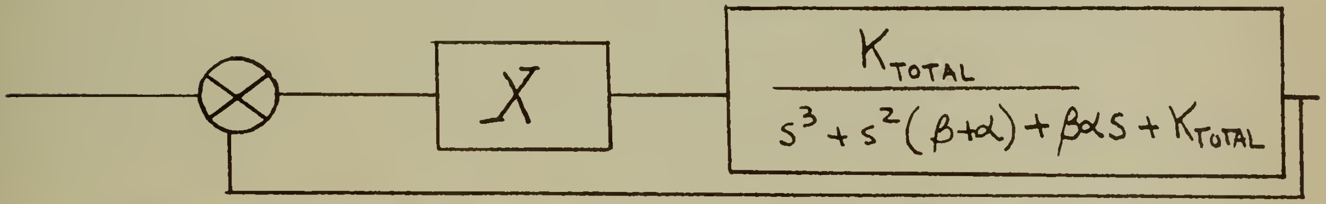
Therefore suppose the values of the various parameters in the initial system are such that the roots are:



and give an unsatisfactory system performance. If this system is to be used something must be done to make its response behave as desired. If a compensator is to be used where shall it be placed? Looking at the physical picture representing our system, points (1) and (2) look like the most probable place to pick off and feed back a signal. Therefore, putting some unknown compensator between these points let's perform our reduction once again.



K
$\frac{K_G}{s(\tau_G + \frac{1}{s})}$
$\frac{K_m}{s(s\tau_m + 1)}$
X



X
$\frac{K_{TOTAL}}{s^3 + s^2(\beta+\alpha) + \beta\alpha s + K_{TOTAL}}$

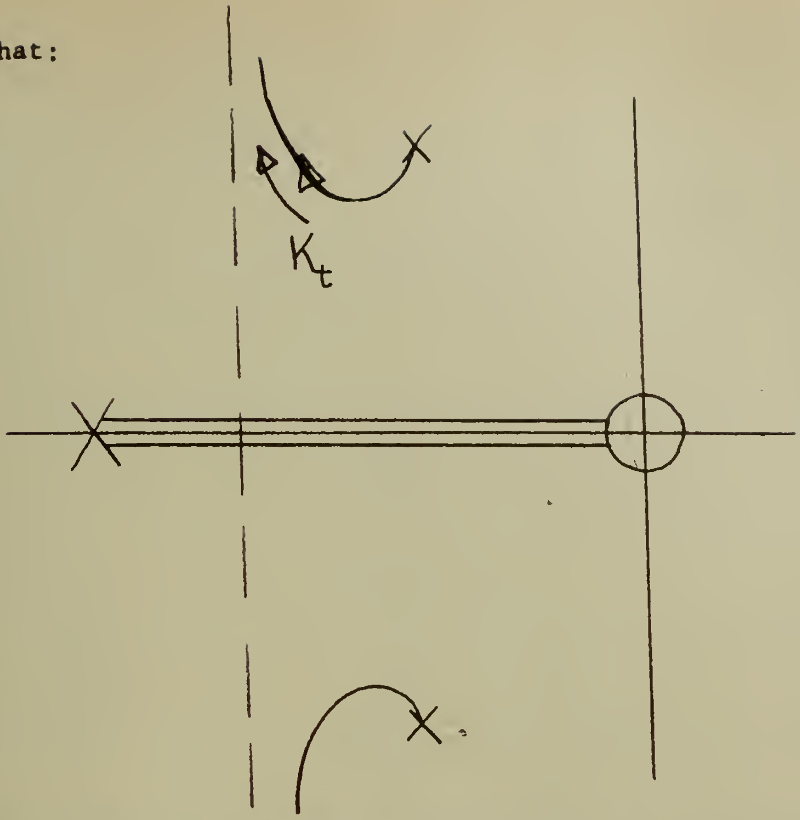
The compensator can be selected and its effects immediately noted to see if it will perform the job required.

That is, suppose $X = K_t S$

then

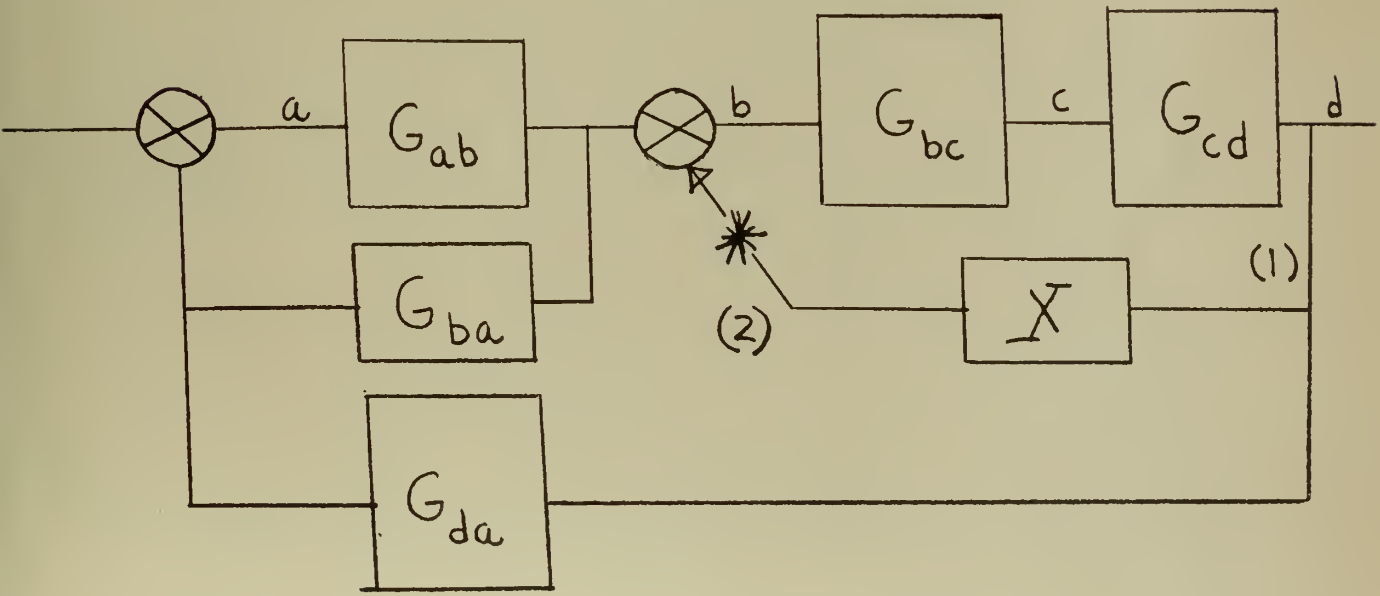
$$1 + KGH = 1 + \frac{K_{TOTAL} K_t S}{s^3 + s^2(\beta+\alpha) + \beta\alpha S + K_{TOTAL}}$$

so that:

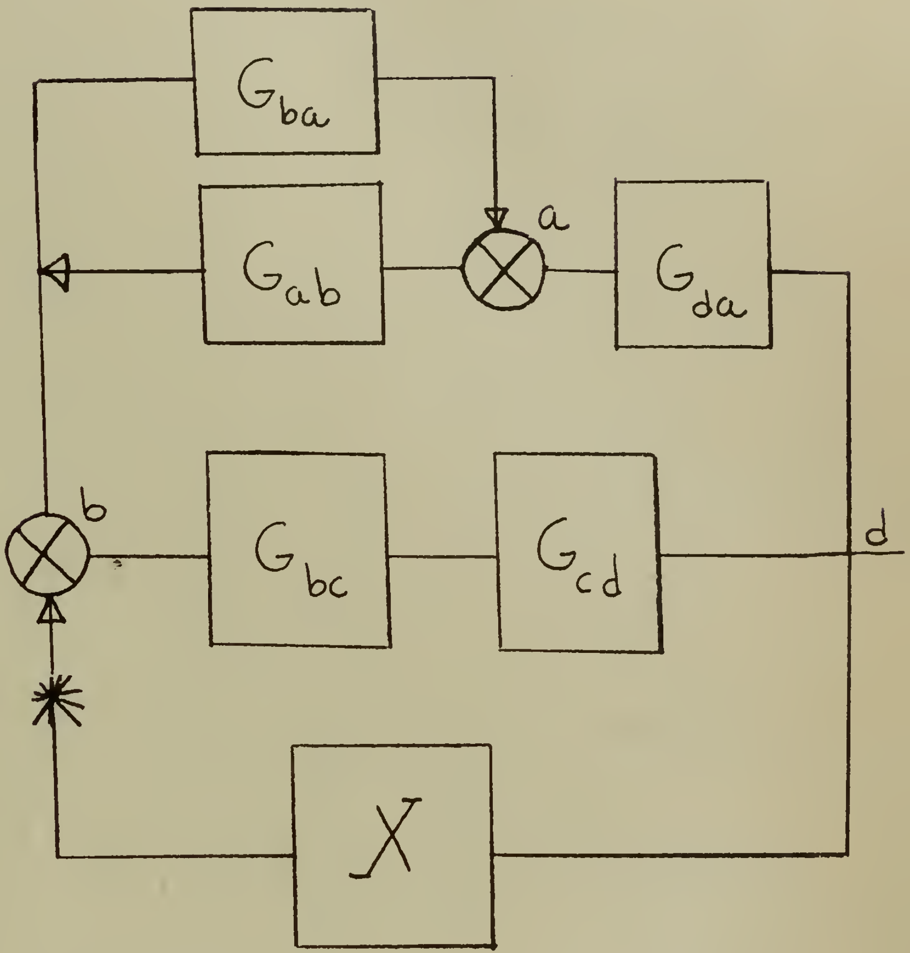


we may obtain good system performance if K_t is the appropriate value. The same analyses can be performed on any compensator selected until one that will meet specifications is found.

Finally consider the following multi-loop system.



Selecting points (1) and (2) as the pick off and feedback points result in the following:



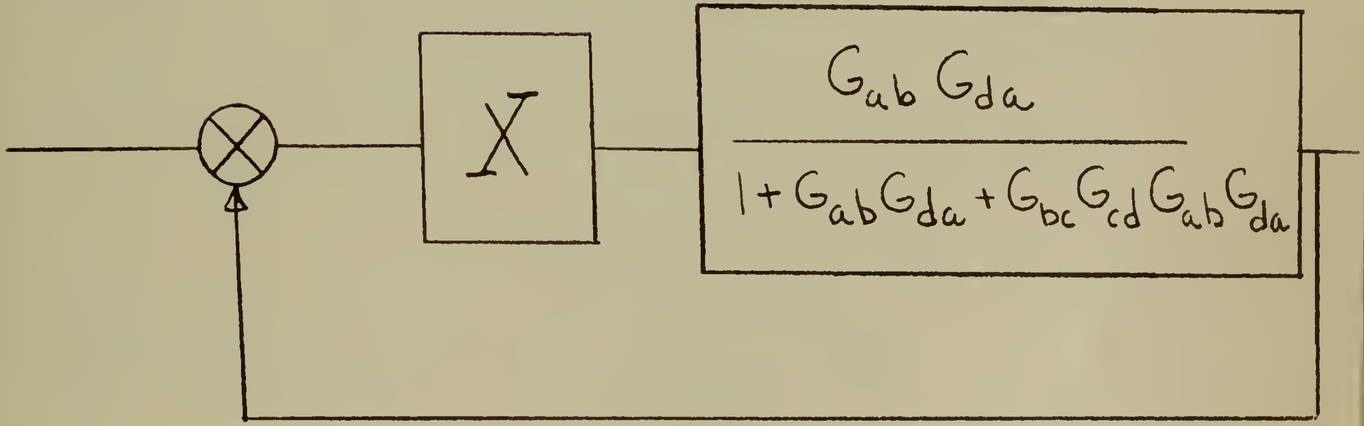
$$G_{\text{EQUIVALENT}\ 1} = \frac{G_{ab}}{1 + G_{ab}G_{ba}}$$

$$G_{\text{EQUIVALENT}\ 2} = \frac{G_{ab}G_{da}}{1 + G_{ab}G_{da}}$$

$$G_{\text{EQUIVALENT TOTAL}} = \frac{\dfrac{G_{ab}G_{da}}{1+G_{ab}G_{da}}}{1+\dfrac{G_{bc}G_{cd}G_{ab}G_{da}}{1+G_{ab}G_{da}}}$$

$$= \frac{G_{ab}G_{da}}{1+G_{ab}G_{da}+G_{bc}G_{cd}G_{ab}G_{da}}$$

Therefore,

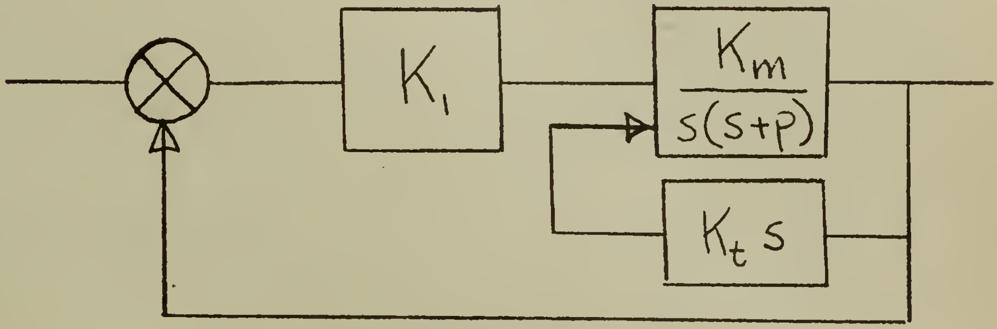

X
$\frac{G_{ab}G_{da}}{1+G_{ab}G_{da}+G_{bc}G_{cd}G_{ab}G_{da}}$

Therefore if the transfer functions were available the initial characteristic equation could be found so that the effect on various compensators can be studied. This then illustrates the general approach taken for any system.

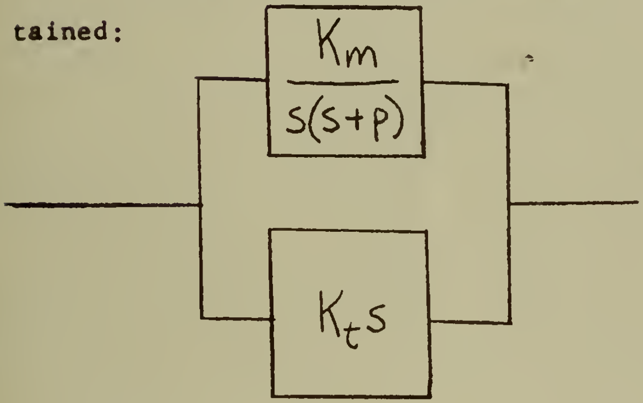
To date, it has just been stated that this manipulation would give the resultant characteristic equation. Now an attempt to prove that this

is so shall be made by comparing the characteristic equation obtained by this method with the one obtained using straight forward block diagram reduction. For example consider the second feedback previously discussed.

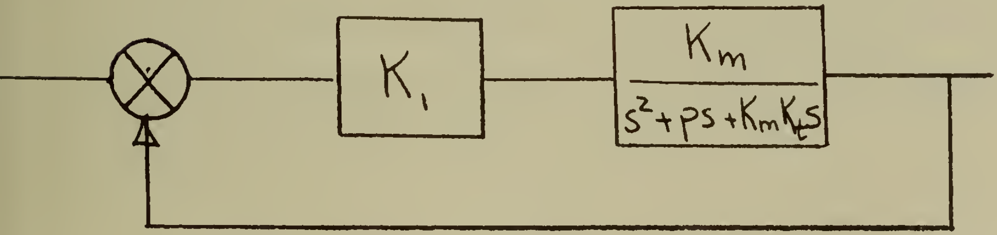
That is



if $X = K_t s$ by simple block diagram reduction the following will be obtained:



$$= G_{EQ} = \frac{K_m}{s^2 + ps + K_m K_t s}$$



while KGH from the block diagram is:

$$\frac{K_1 K_m}{s^2 + ps + K_m K_t s}$$

this gives

$$1 + KGH = 0 = 1 + \frac{K_1 K_m}{s^2 + ps + K_m K_t s}$$

$$= \frac{s^2 + ps + K_m K_t s + K_1 K_m}{s^2 + ps + K_m K_t s}$$

but from our previous derivation

$$1 + KGH = 0 = 1 + \frac{K_m K_t s}{s^2 + ps + K_1 K_m}$$

$$= \frac{s^2 + ps + K_m K_t s + K_1 K_m}{s^2 + ps + K_1 K_m}$$

The numerators are identical and are the characteristic equation.

This is exactly the same result obtained employed the new reduction technique explained previously. Just one such proof will be presented here since in the next chapter when we deal strictly with acceleration feedback, the characteristic equation shall be obtained by several methods. Once again however the technique presented here is by far faster and more adapted for use in analyzing various compensating devices and their effect on the overall system.

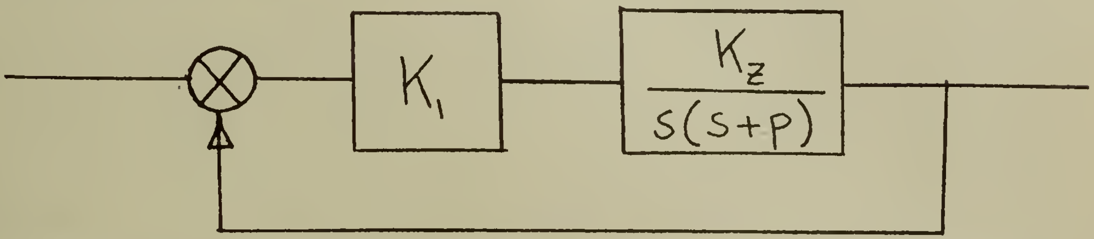
CHAPTER TWO

THE EFFECTS OF ACCELERATION FEEDBACK UPON A SECOND ORDER SYSTEM

The design of a feedback control system involves a compromise between the magnitude of the steady state error and the degree of stability. After the first analysis of the system, it may be found that the system has too great a steady state error (possibly K_v is too small); or the steady state errors may be within specifications, but the system is too close to instability (i.e., the damping ratio is too small). In either case the system must be provided with compensation in order to function in accordance with certain specifications. This chapter shall deal primarily with the utilization of acceleration feedback as the means of compensation.

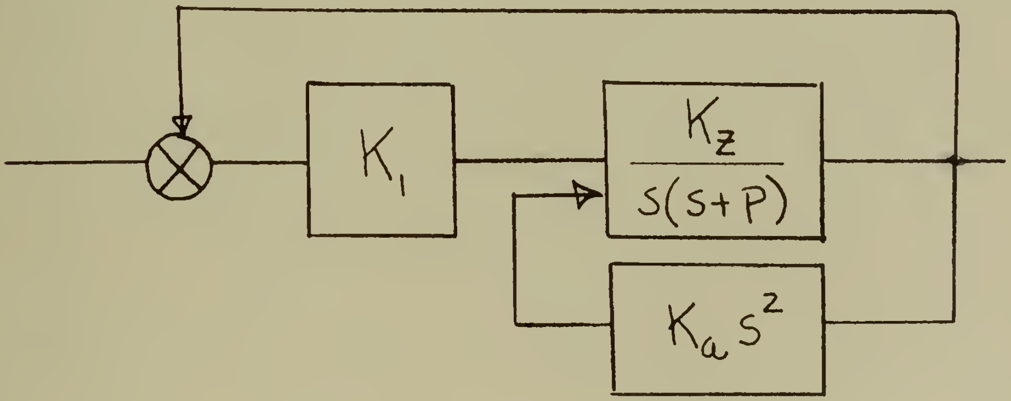
However in performing an investigation utilizing acceleration feedback, let's select a practical type problem - one that is of some importance in the application of certain systems. One common problem arises when the roots of a system provide a damping ratio that is entirely too low causing excessive oscillation. Since in most cases, the characteristics of the particular devices utilized are fixed (i.e., the gain and transfer function that represent these pieces of equipment often depends upon internal make up or construction and cannot be changed), acceleration feedback is then one way in which a better system response, can be obtained. As a matter of fact it will be seen that utilizing this type of feedback compensation the engineer need only select the desired location of the roots of the system, and the value of gain to be used in the feedback loop as well as the compensator parameters will be readily available.

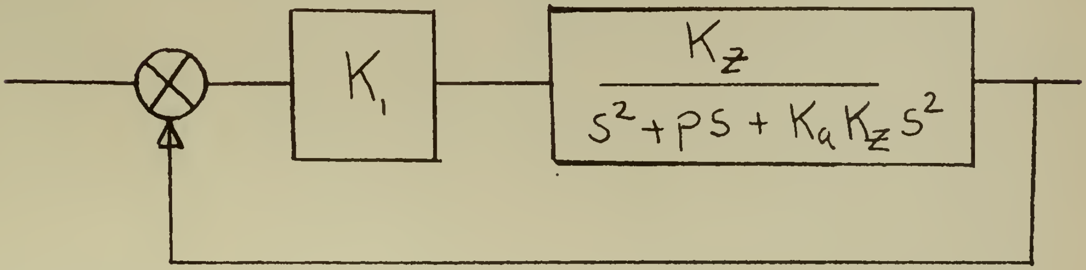
Let us consider the following second order system, with inherent values of gain K_1 K_z such that the roots obtained possess a damping ratio of the order 0. 1 or less.

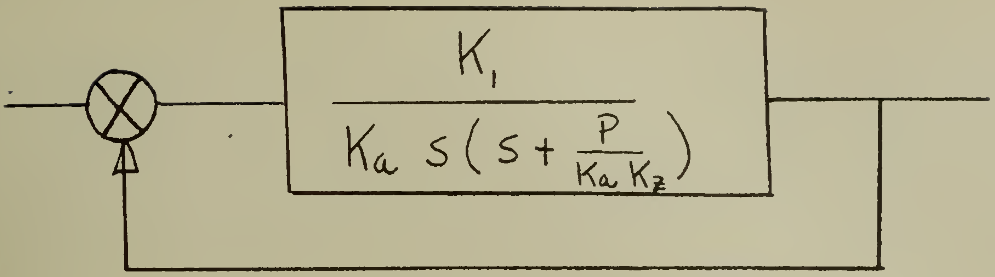


New in the discussion of acceleration feedback, pure acceleration feedback as a possibility will be eliminated because the (ζ) that results even when it is used is still too low to be of practical value. This can be proven by the following illustrations.

A) To prove pure acceleration feedback provides a Pole and zero configuration that is undesirable







Therefore

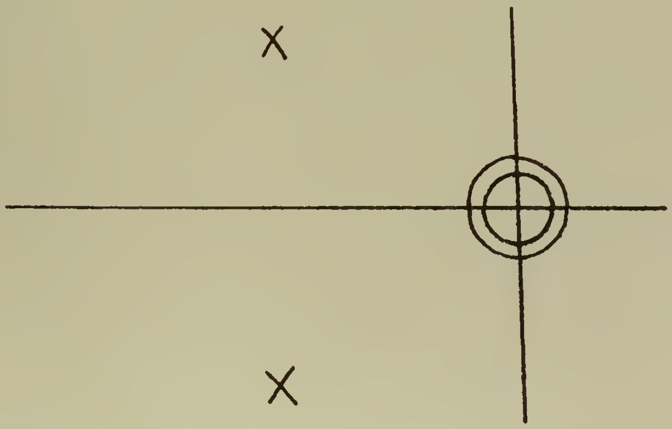
$$\frac{\theta_c}{\theta_r} = \frac{K_1 K_z}{s(s+p)+K_z K_a s^2 + K_1 K_z}$$

and the characteristic equation is

$$1+KGH = s(s+p)+K_z K_a s^2 + K_1 K_z = 0$$

so that

$$\frac{K_a K_z s^2}{s^2+ps+K_1 K_z} = -1$$



Two examples to show the effects of pure acceleration feedback will now be presented.

EXAMPLE ONE

$K_1 = 1.0 \quad K_z = 10 \quad K_a = 5 \; \xi \; p = 2.0$

THEREFORE $\quad 1 + KGH = 1 + \frac{50s^2}{s^2 + 2s + 10} = 0$

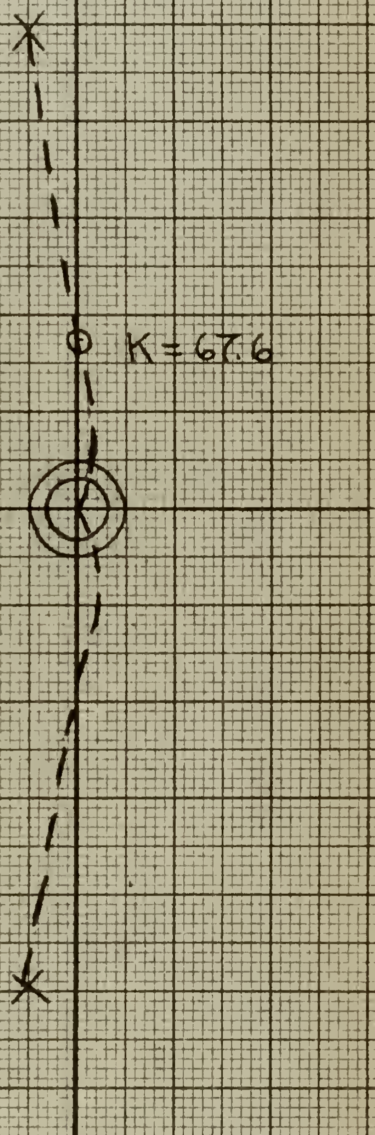
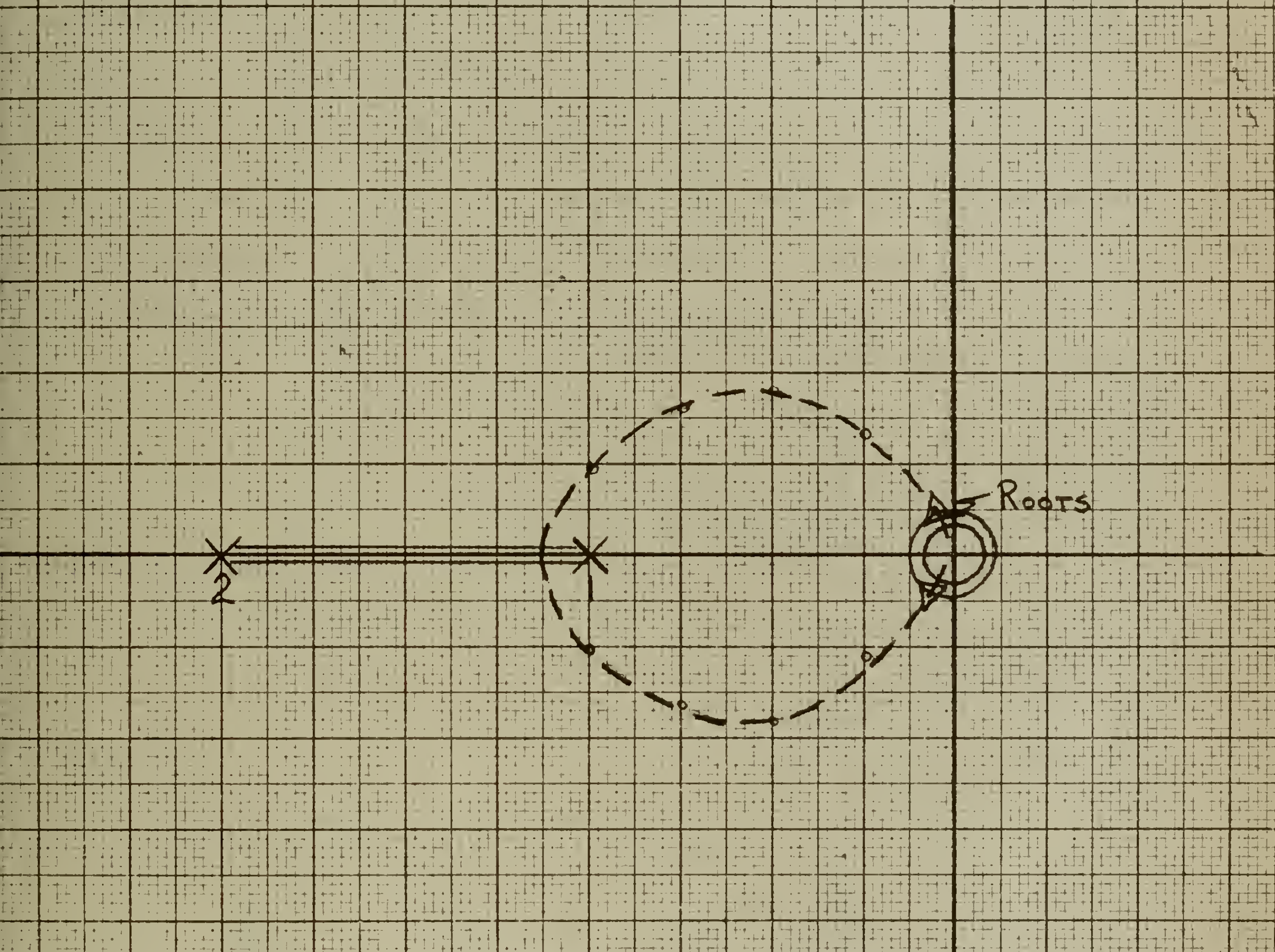


Fig. #2-1. EXAMPLE #1 - TO ILLUSTRATE THE EFFECTS OF PURE ACCELERATION FEEDBACK

EXAMPLE TWO

$K_1 = (1.0)2, \ K_2 = 1.0 \quad K_a = 10 \quad \& \ p = 3.0$

THEREFORE $\quad 1 + KGH = 1 + \dfrac{10s^2}{s^2 + 3s + 2}$

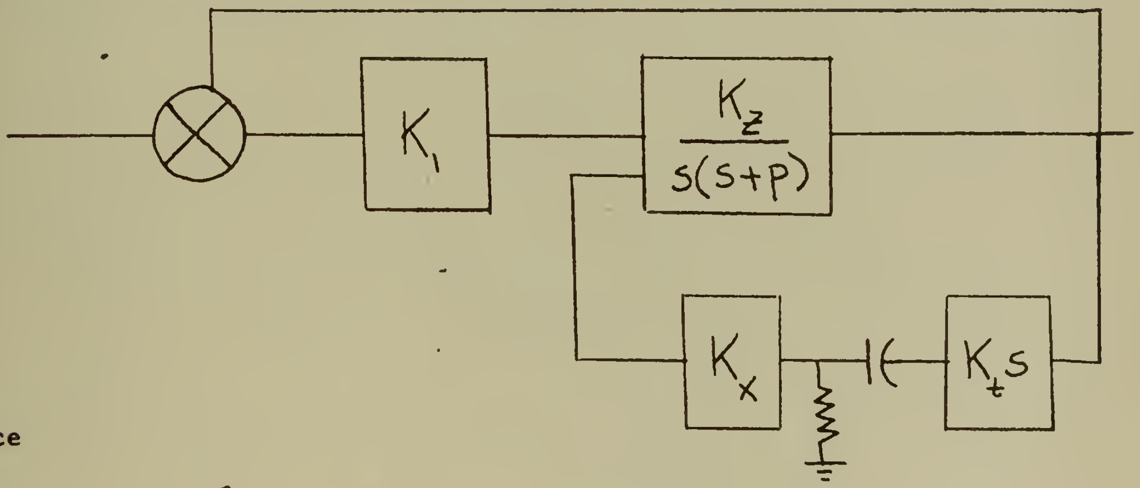


NOTICE - IN BOTH EXAMPLES
SYSTEM IS EITHER UNSTABLE
OR HAS A VERY LOW ζ

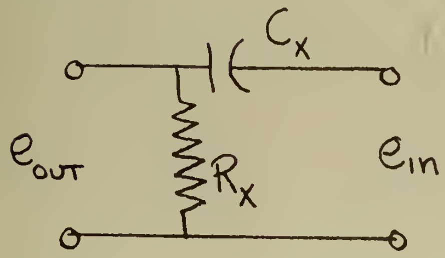
Figure 2-2. EXAMPLE #2 ILLUSTRATING EFFECTS OF PURE ACCELERATION FEEDBACK

Therefore, with pure acceleration eliminated as a possibility, what exactly is meant by acceleration feedback? How else can it be obtained? It will be noted that first derivative feedback can almost always be obtained, either from the simplest case when a tachometer pickoff can be used to feed the signal back to the more complex when filters are used to obtain it. (In this regard, K_tS is very closely approximated if the pole of the filter is very far out on the negative real axis- the analogue computer is a common illustration of a device which does utilize this principle to attain first-derivative feedback.) Therefore by putting this first derivative signal into an appropriate filter, the desired transfer function of the feedback loop can be obtained.

Taking the general case for a second order system



since



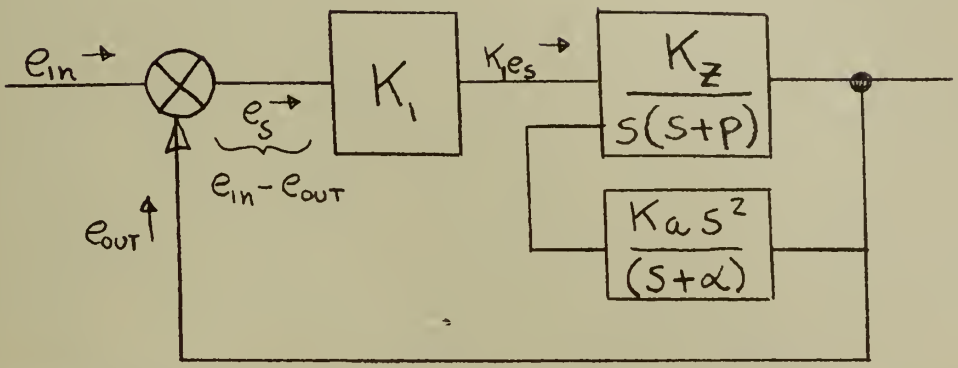
$$e_{OUT} = \frac{R_x}{R_x + \frac{1}{C_x S}} e_{in}$$

$$e_{OUT} = \frac{S}{S+\alpha} e_{in}$$

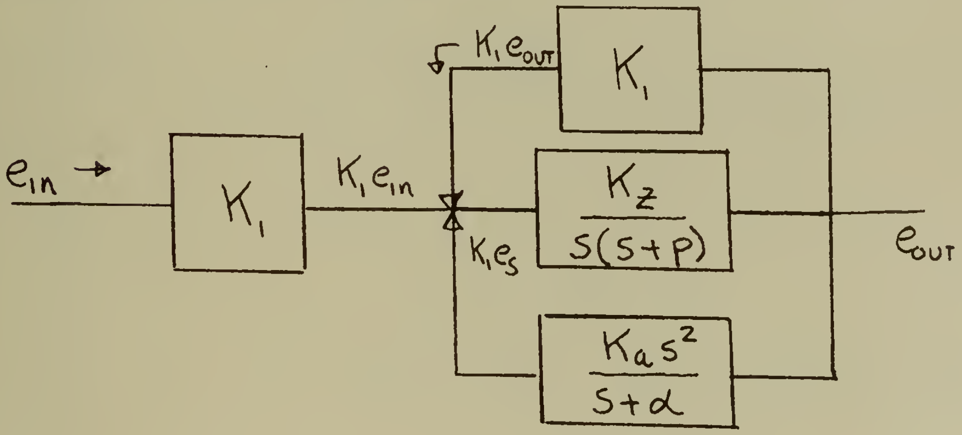
WHERE $\alpha = \frac{1}{R_x C_x}$

if we also define $e_{in} = K_t s$ & $K_a = K_x K_t$

Then we have $e_{OUT} = \dfrac{K_t s^2}{s+\alpha}$



Now reduce entire system to an equivalent block in cascade with the compensator



combining K_1 & $\dfrac{K_z}{s(s+p)}$

$$G \text{ equivalent} = \frac{\dfrac{K_z}{s(s+p)}}{1 + \dfrac{K_1 K_z}{s(s+p)}} = \frac{K_z}{s^2 + ps + K_1 K_z}$$

Therefore for the entire network

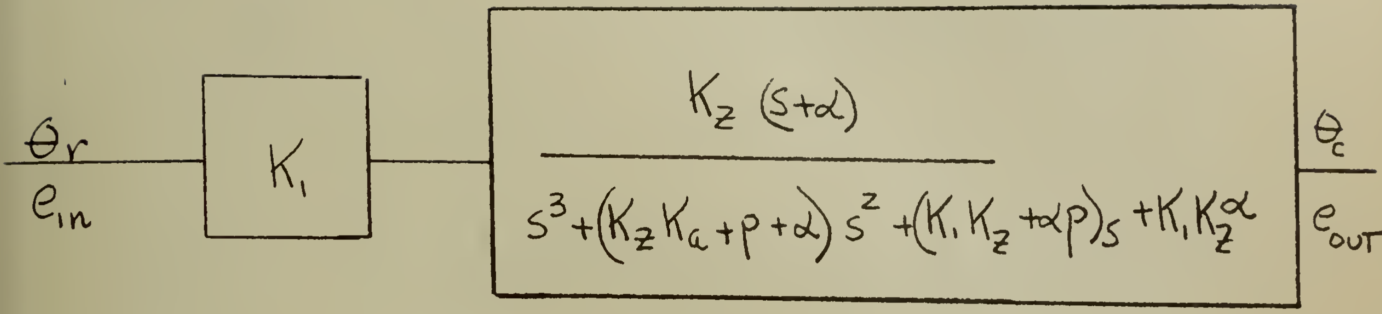
$$G_{Total} = \frac{\dfrac{K_z}{s^2 + ps + K_1 K_z}}{1 + \dfrac{K_z K_a s^2}{(s+\alpha)(s^2 + ps + K_1 K_z)}}$$

$$= \frac{K_z (s+\alpha)}{(s+\alpha)(s^2 + ps + K_1 K_z) + K_z K_a s^2}$$

$$= \frac{K_z (s+\alpha)}{s^3 + (K_a K_z + p + \alpha) s^2 + (K_1 K_z + \alpha p) s + K_1 K_z \alpha}$$

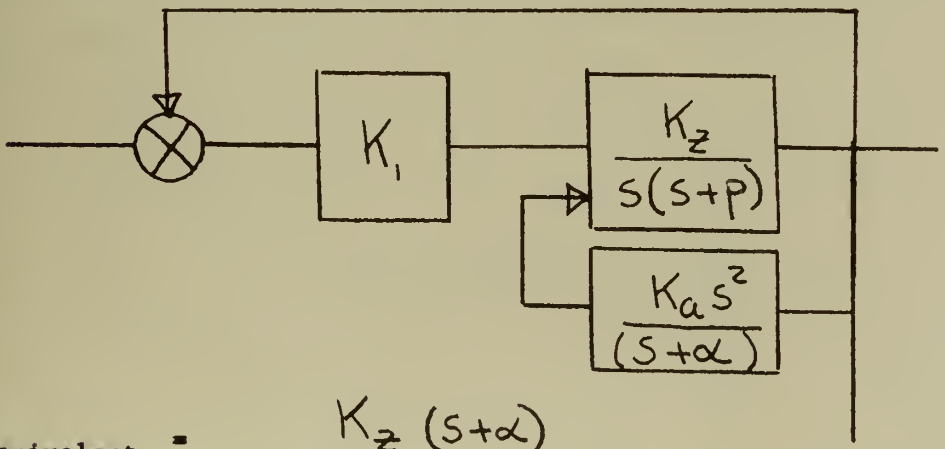
and characteristic equation is

$$1 + KGH = s^3 + (K_a K_z + p + \alpha) s^2 + (K_1 K_z + \alpha p) s + K_1 K_z \alpha = 0$$


θ_r
e_{in}
K_1
$\dfrac{K_z(s+\alpha)}{s^3+(K_zK_a+p+\alpha)s^2+(K_1K_z+\alpha p)s+K_1K_z\alpha}$
θ_c
e_{OUT}

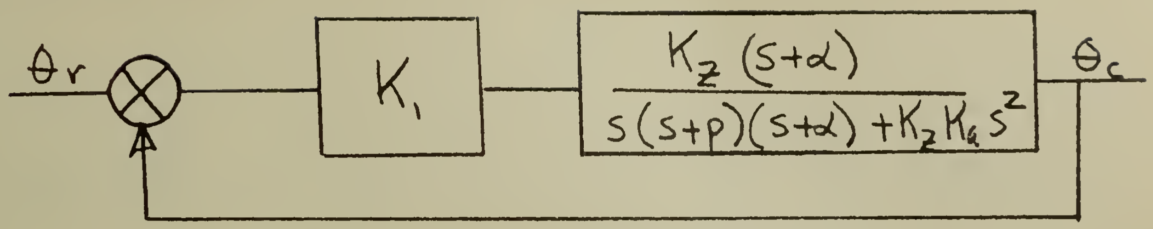
Therefore (1).
$$\frac{\theta_c}{\theta_r} = \frac{K_1K_z(s+\alpha)}{s^3+(K_zK_a+p+\alpha)s^2+(K_1K_z+\alpha p)s+K_1K_z\alpha}$$

Another way to look at problem of reduction:



$$G_{Equivalent} = \frac{K_z(s+\alpha)}{s(s+p)(s+\alpha)+K_zK_as^2}$$

Therefore



$$\frac{\theta_c}{\theta_r} = \frac{K_1 K_z (s+\alpha)}{s(s+p)(s+\alpha) + K_z K_a s^2 + K_1 K_z (s+\alpha)}$$

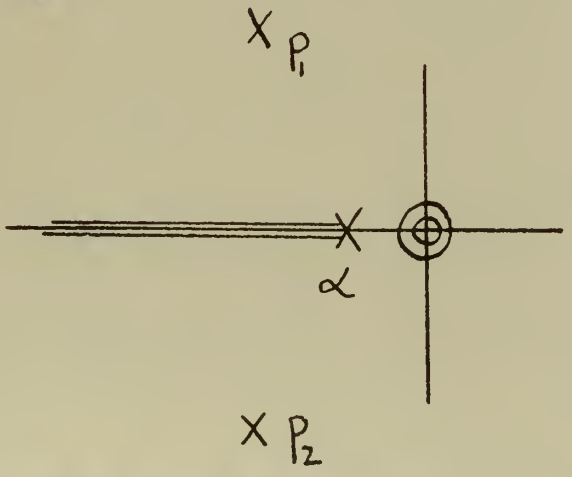
This checks with (1)

Therefore the characteristic equation of the system can be written:

$$1 + KGH = s(s+p)(s+\alpha) + K_z K_a s^2 + K_1 K_z (s+\alpha) = 0$$

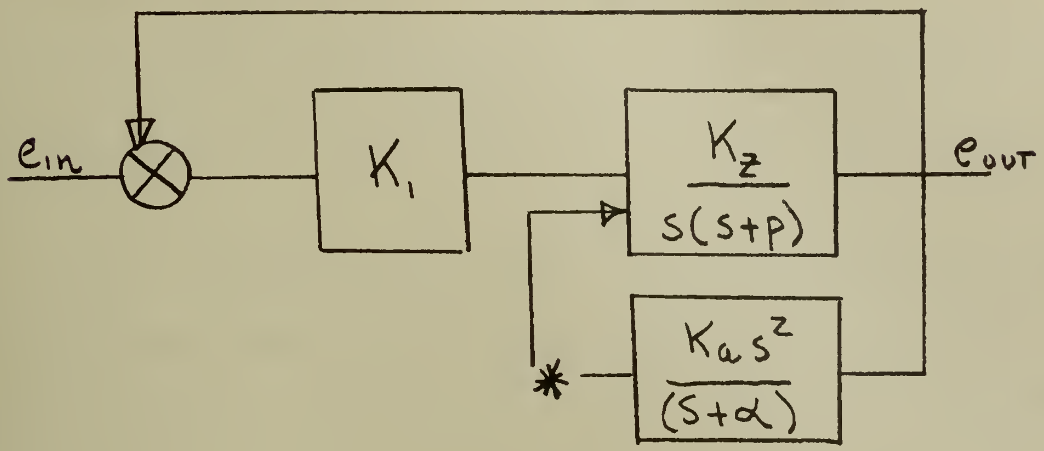
$$\therefore \quad \frac{K_z K_a s^2}{(s+\alpha)(s^2 + ps + K_1 K_z)} = -1$$

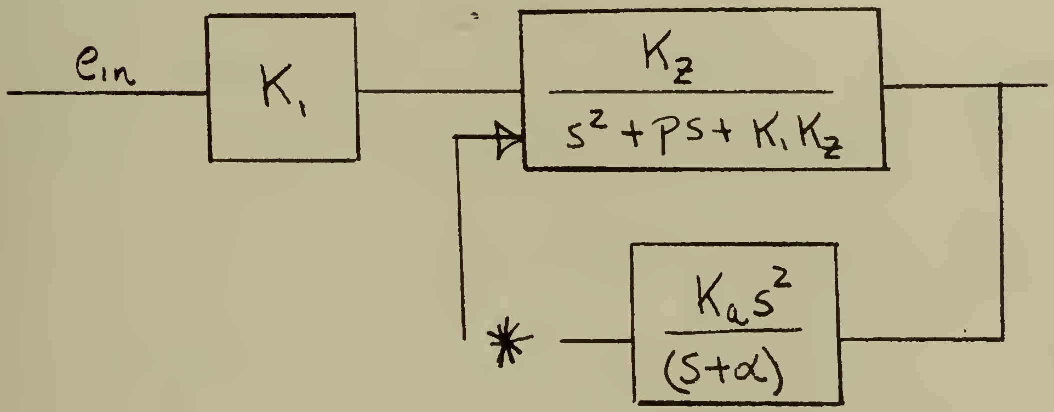
Thus it can be seen we do obtain an additional pole determined by the parameters of our filter.



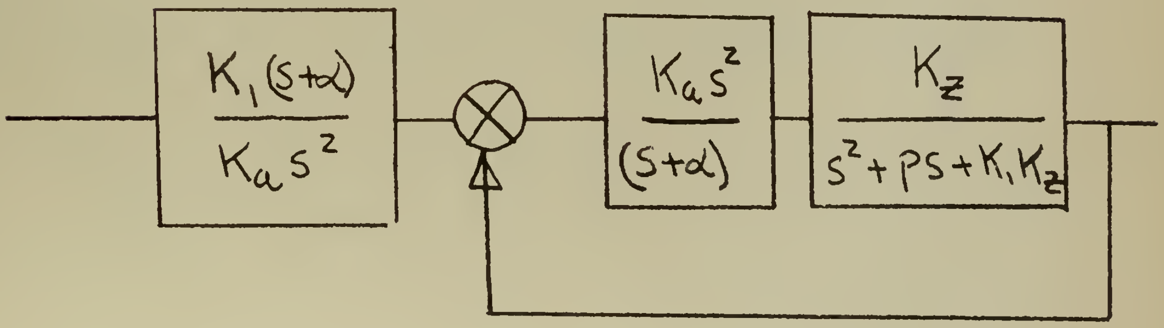
The above root locus diagram illustrates variable pole and zero locations.

Finally employing the reduction method discussed in Chapter One, the compensator will be put directly in cascade with network





then

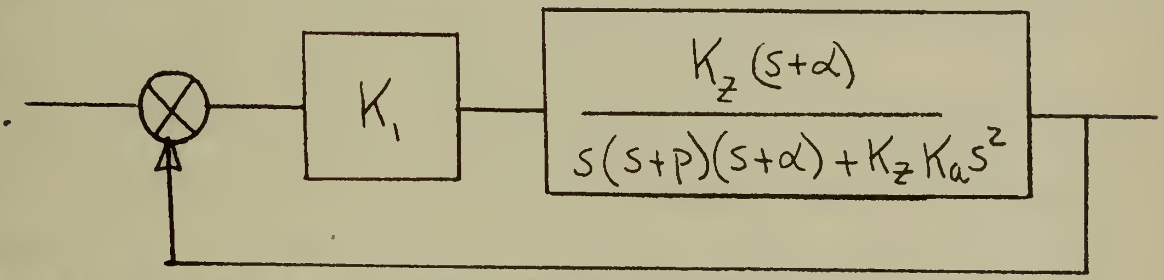

$\frac{K_1(s+\alpha)}{K_a s^2}$
$\frac{K_a s^2}{(s+\alpha)}$
$\frac{K_2}{s^2+ps+K_1K_2}$

The characteristic equation is still:

$$1 + \frac{K_a K_z s^2}{(s^2 + ps + K_1 K_z)(s+\alpha)} = 0$$

It will be noted that in presenting the general case under study for the second order system several methods of reduction were employed to obtain the resultant characteristic equation. This was done to further illustrate the ease of manipulation of the method presented in Chapter One over the others and to prove that the characteristic equation is the same no matter what method of reduction is used.

Before continuing, let's look at the value of the velocity error coefficient K_v to see what effects this type of compensation has upon it since



was obtained using the block diagram method. Based upon a system which has unity feedback (H = 1) the velocity error coefficient is defined

$$K_v = \lim_{s \to 0} s\,KG(s)$$

Therefore

$$K_v = \lim_{s \to 0} s \frac{K_1 K_z (s+\alpha)}{s^3 + (K_z K_a + p + \alpha)s^2 + p\alpha s}$$

$$= \frac{K_1 K_z \alpha}{p\alpha} = \frac{K_1 K_z}{P}$$

Hence, it can be seen that for any particular second order system having a gain K_1K_z and a pole P, the K_v will be constant regardless what the gain in the feedback loop (K_a) is.

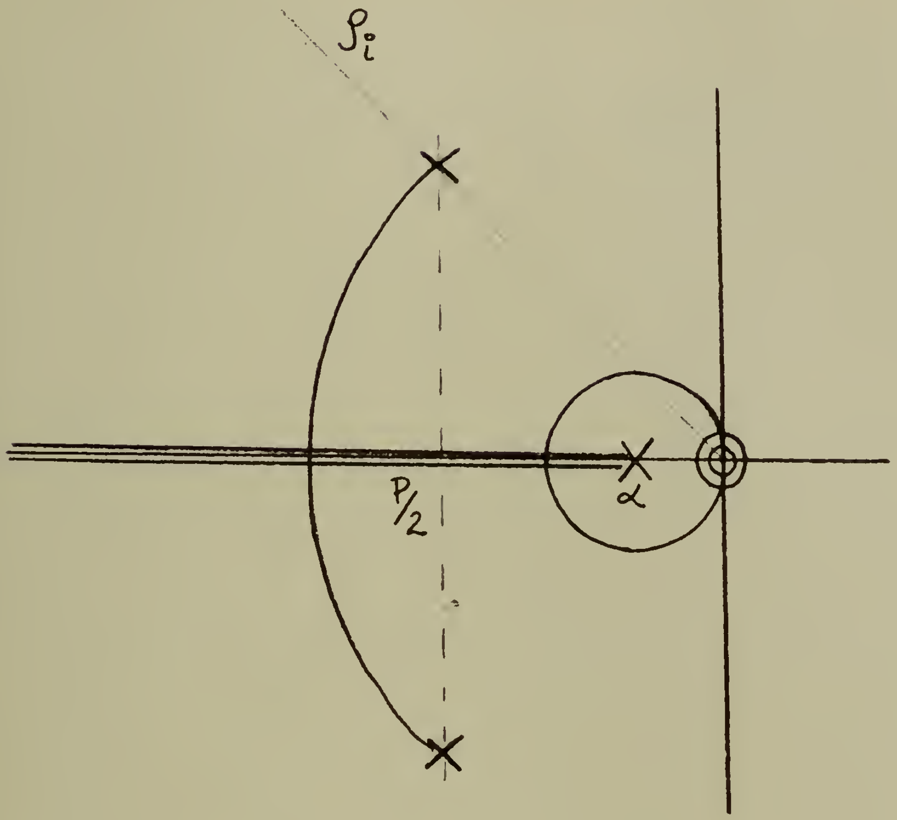
Having obtained the characteristic equation of the entire system when this type of compensation is employed the following questions then must be answered. Exactly what effect does this type of device have on the system? Does it improve its performance and if so, in what way? Is there a method of generalizing the effects of this type of compensation? And finally, will it be of practical interest to the control engineer?

Before these questions can be answered however the resultant root locus must be analyzed so that it may become clear exactly where the new roots of the system may be placed using acceleration feedback. Knowing the location of the roots the transient response can be obtained to see if the operation has been improved.With this information on hand the remaining questions can be answered rather simply.

Therefore if the characteristic equation of the resultant system is

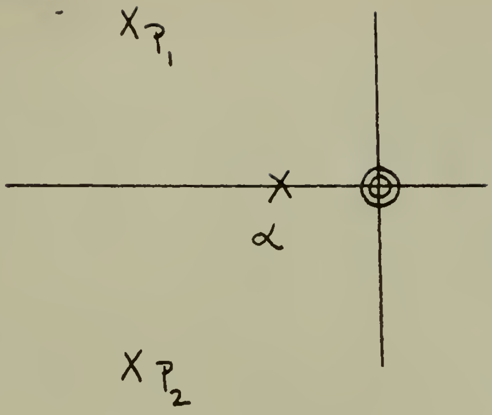
$$1 + \frac{K_z K_a s^2}{(s^2 + ps + K_1 K_z)(s+\alpha)} = 0$$

the root locus will take the following form (See Appendix A)



As illustrated the root locus consists of pure circles and semi circles (especially if the initial roots have a damping ratio that is relatively low (say 0.1) or less and the location of the compensator pole is 0.05 p or less). If the initial roots possess larger and larger damping ratios the root locus will still be very close to the pure circles and semi circles. However if it is desired to have the compensator pole further out on the negative real axis the root locus deviates somewhat from the pure circles and semi circles until a critical value is reached at which the root locus doesn't touch the real axis at all. This fact is illustrated also from root locus plots taken utilizing the Esiac plotter.

Using Esiac - obtain plots for



Case a. P_1 & P_2 $= -1 \pm j1$

$\alpha = .10$

Case b. P_1 & P_2 $= -1 \pm j1$

$\alpha = .30$

Case c. P_1 & P_2 $= -1 \pm j1$

$\alpha = .50$

Case d. P_1 & P_2 $= -1 \pm j1$

$\alpha = .40$

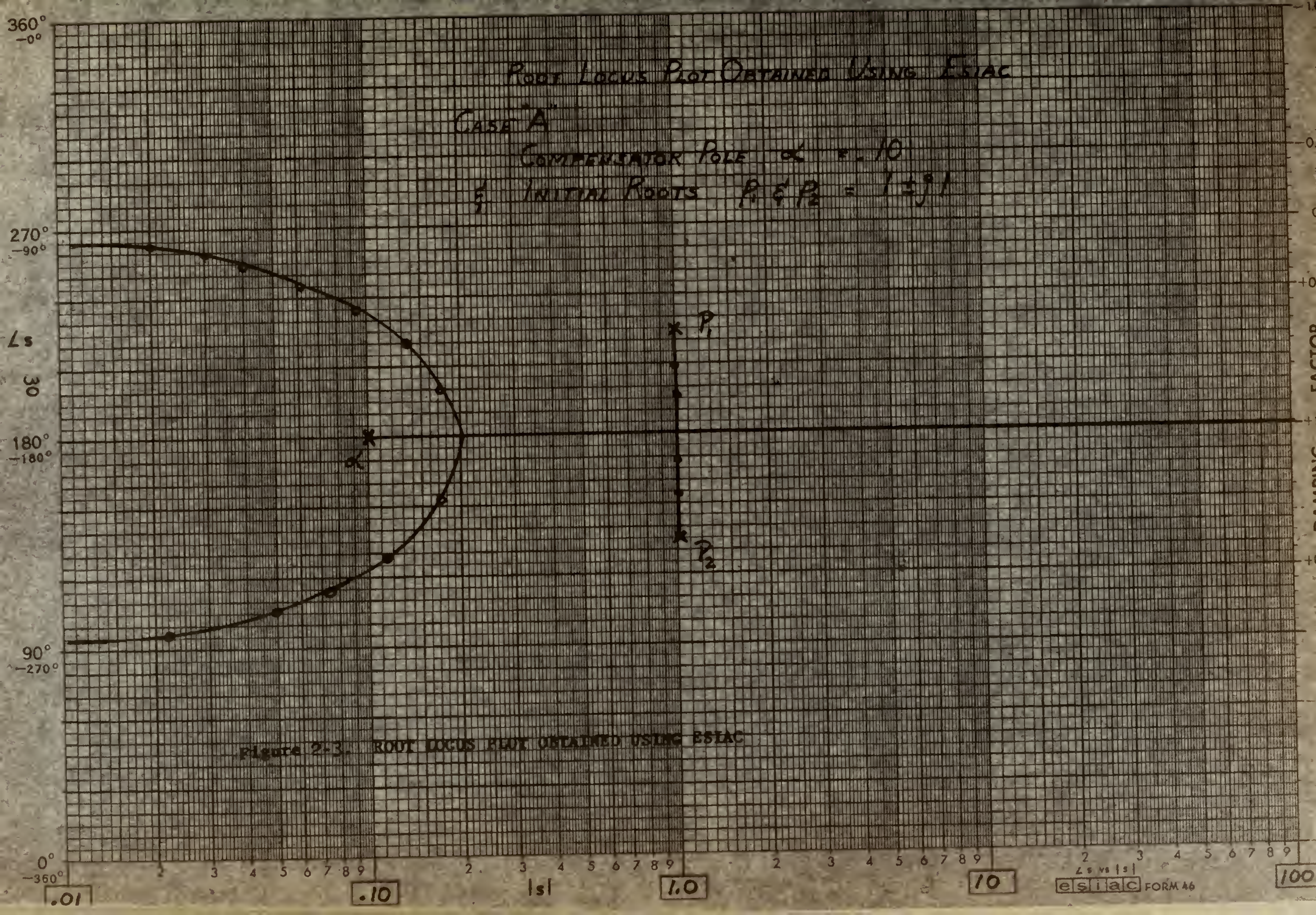


Figure 2-3. ROOT LOCUS PLOT OBTAINED USING ESIAC

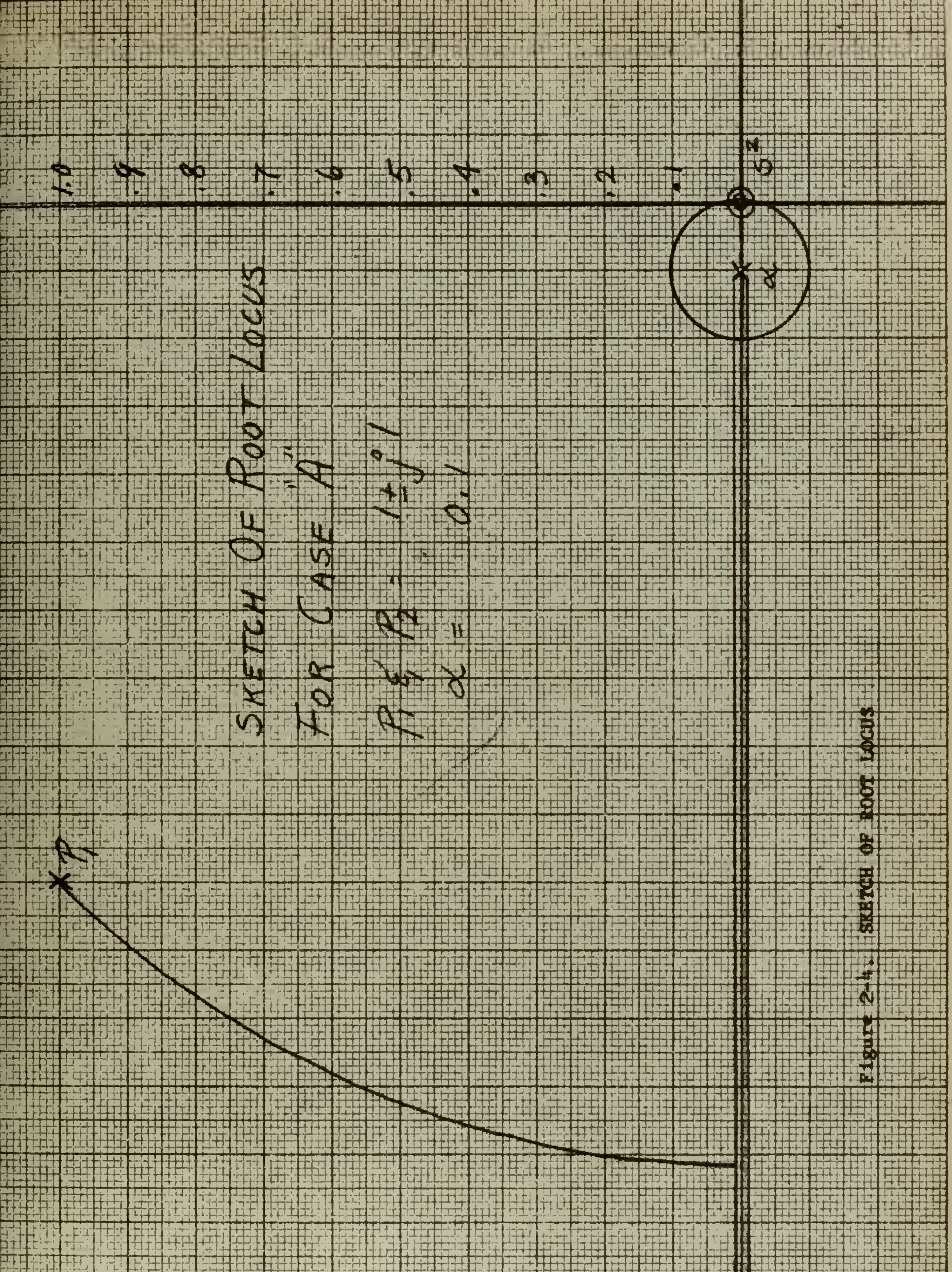


Figure 2-4. SKETCH OF ROOT LOCUS

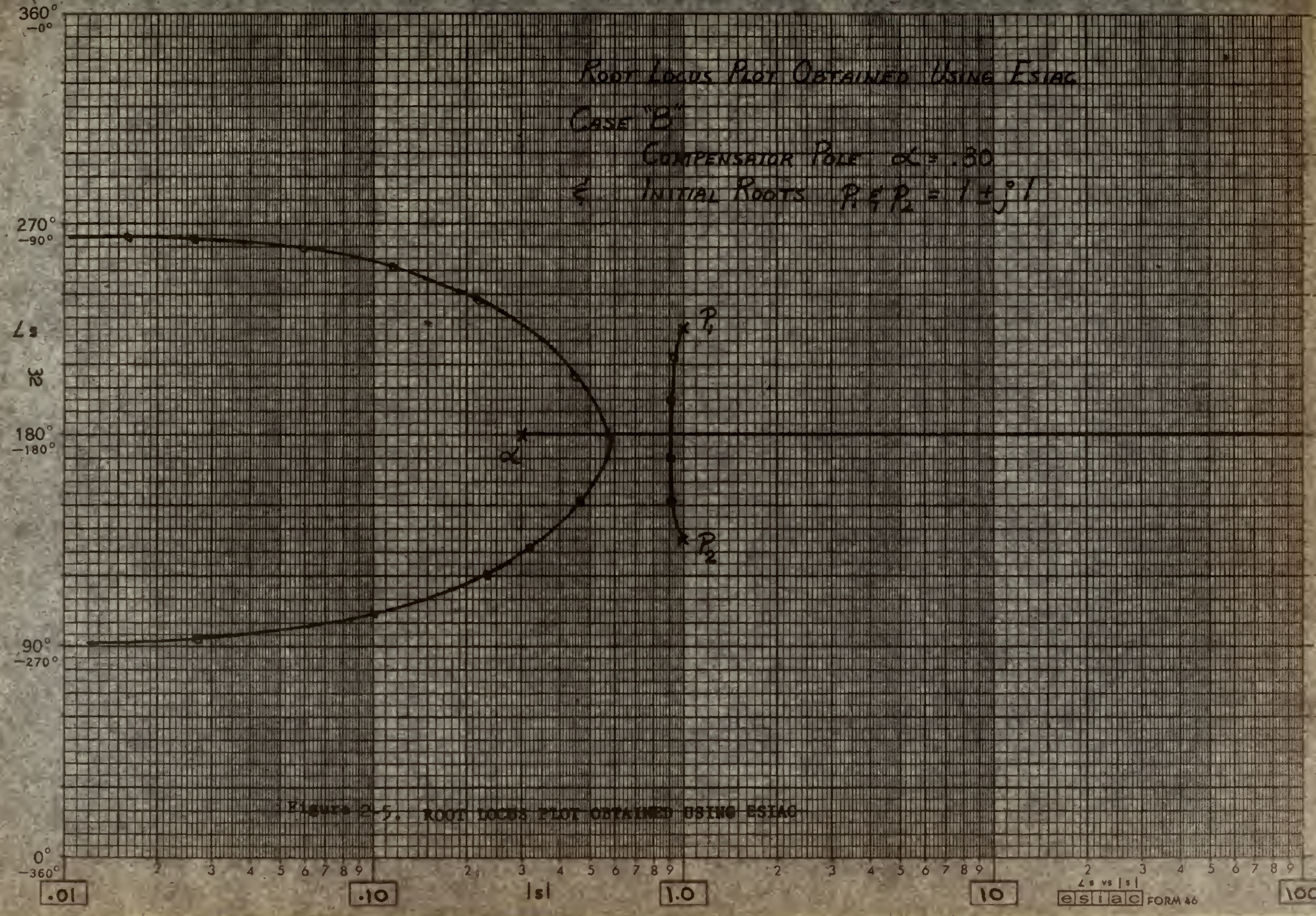


Figure 2-5. ROOT LOCUS PLOT OBTAINED USING ESIAC

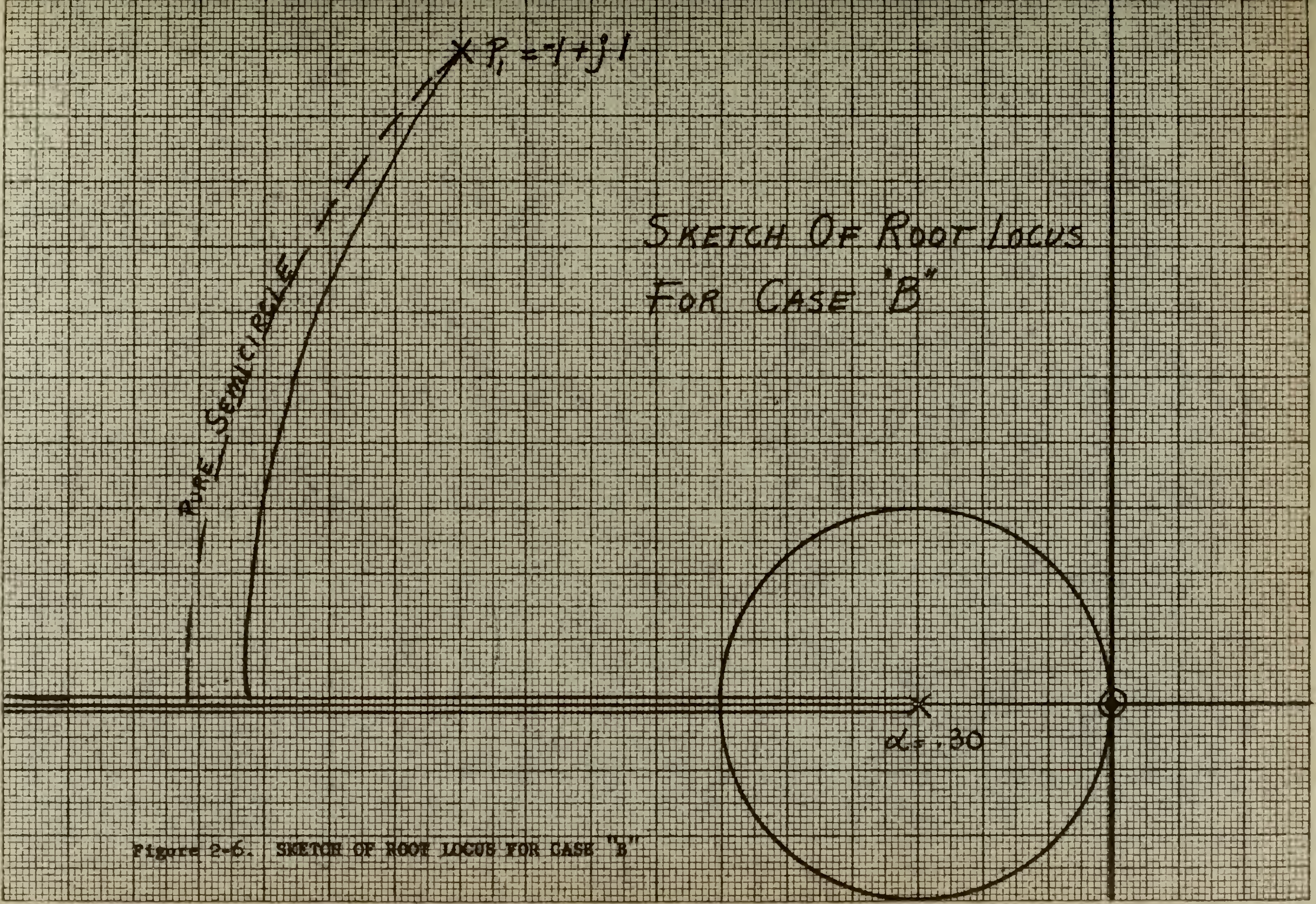


Figure 2-6. SKETCH OF ROOT LOCUS FOR CASE "B"

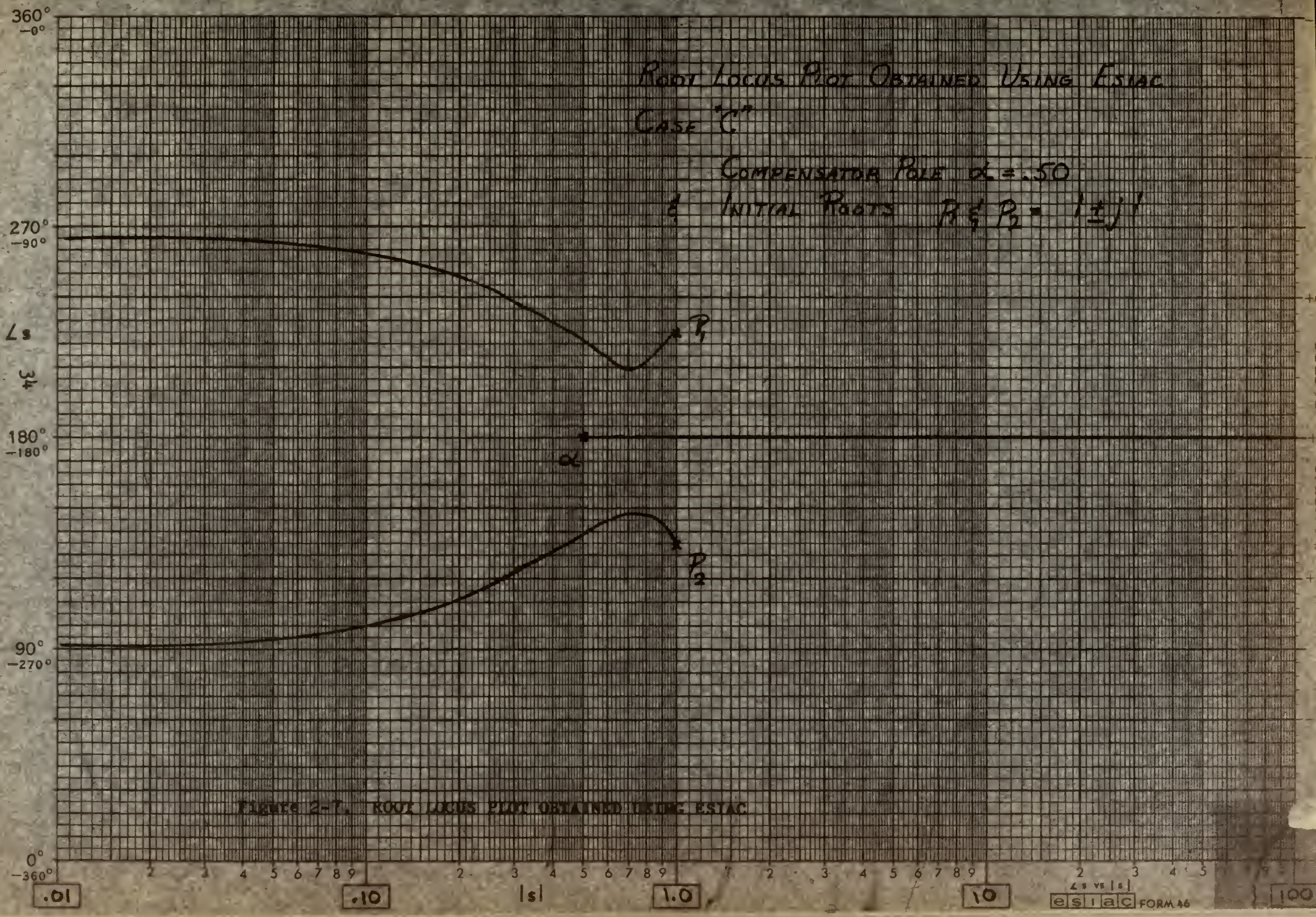


Figure 2-7. ROOT LOCUS PLOT OBTAINED USING ESIAC

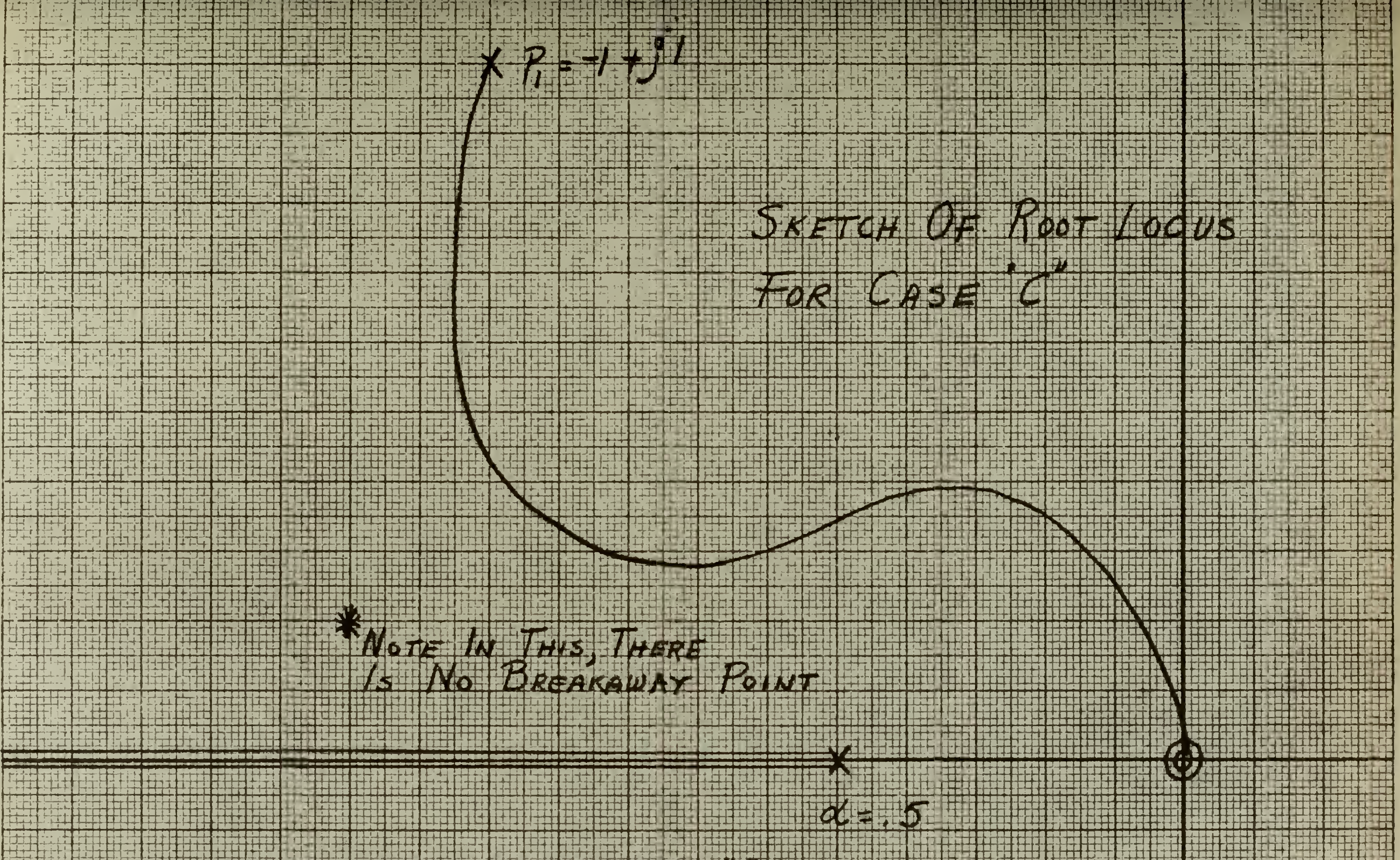
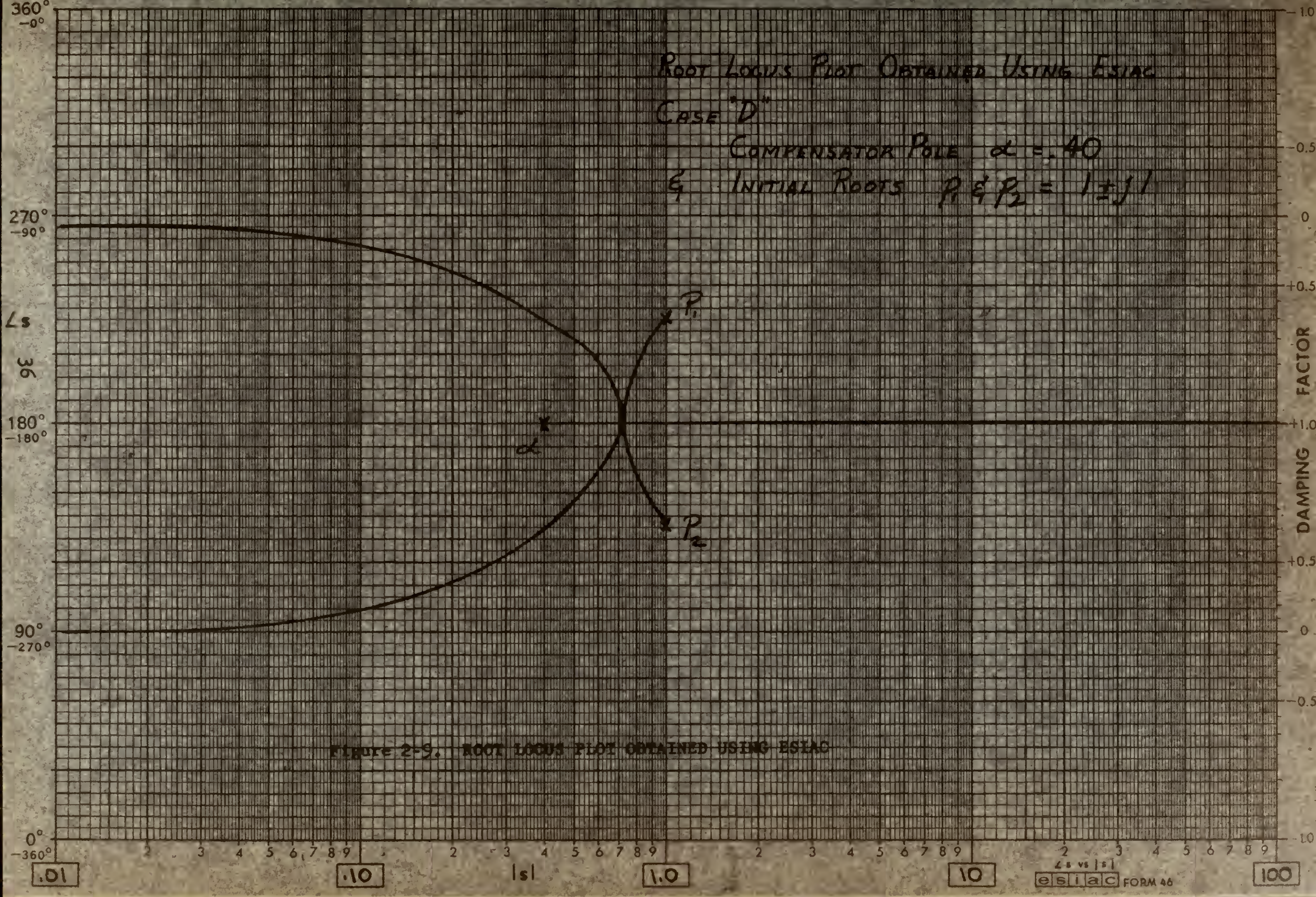


Figure 2-8. SKETCH OF ROOT LOCUS FOR CASE "C"



Root Locus Plot Obtained Using Esiac

Case "D"

Compensator Pole $\alpha = .40$

& Initial Roots P_1 & $P_2 = 1 \pm j1$

Figure 2-9. ROOT LOCUS PLOT OBTAINED USING ESIAC

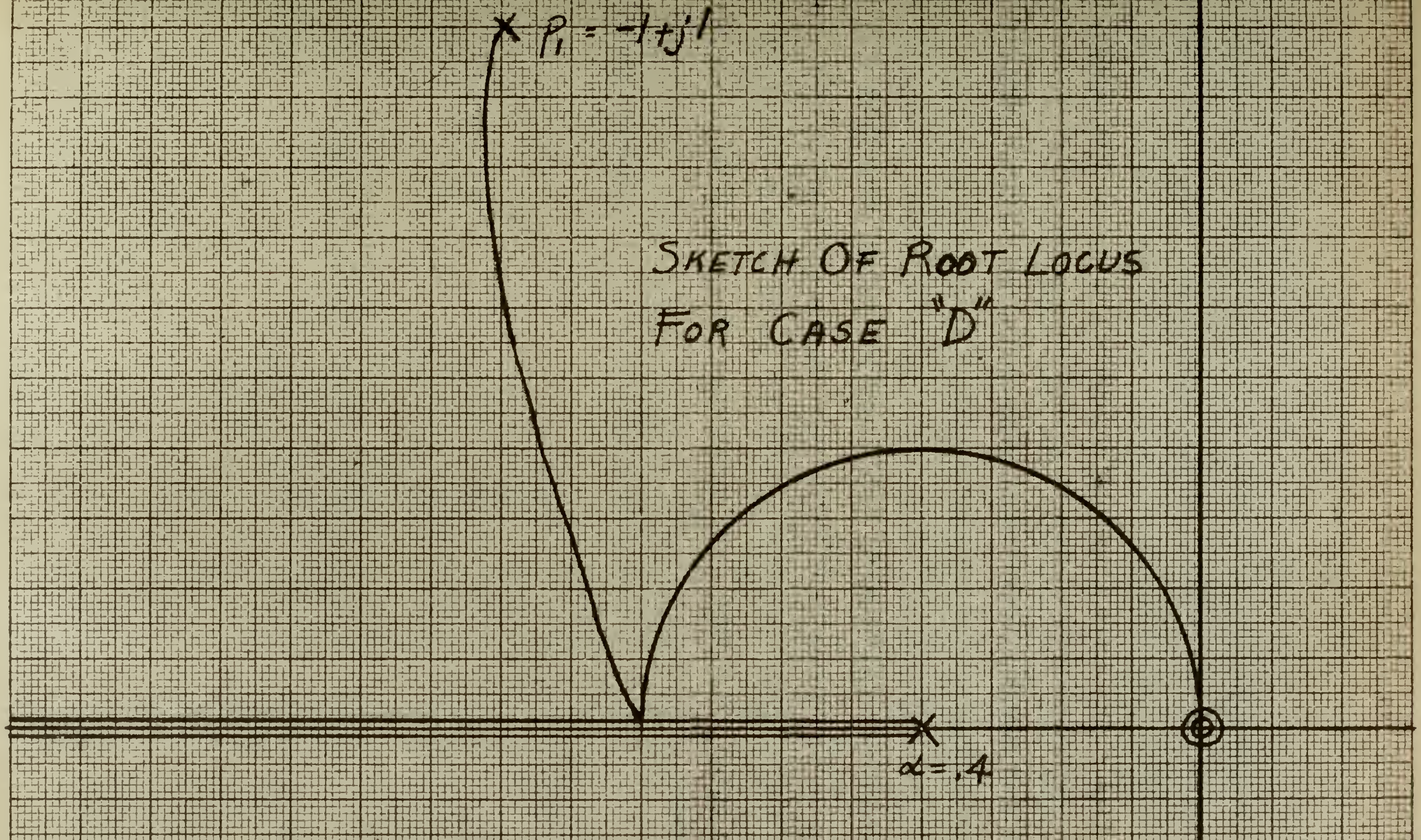
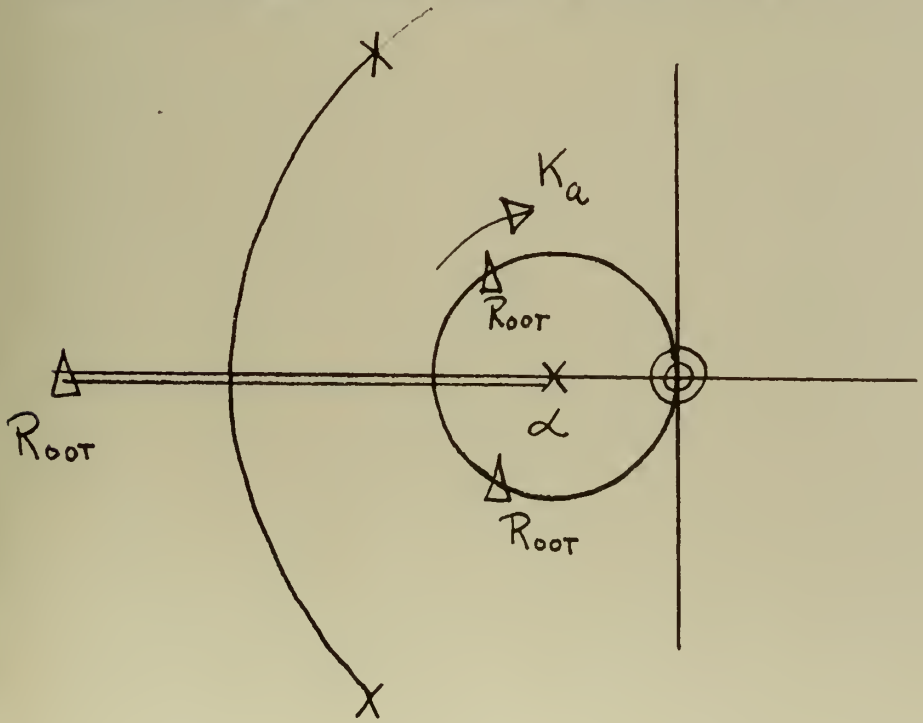


Figure 2-10. SKETCH OF ROOT LOCUS FOR CASE "D"

It then has been noted exactly what the shape of the root locus will be for various compensator poles and initial system roots. It is obvious however that if the system has roots initially having a large damping ratio, there will be little need for compensation. Therefore for low initial damping ratios, the root locus will be closely approximated by pure circles and semi circles.

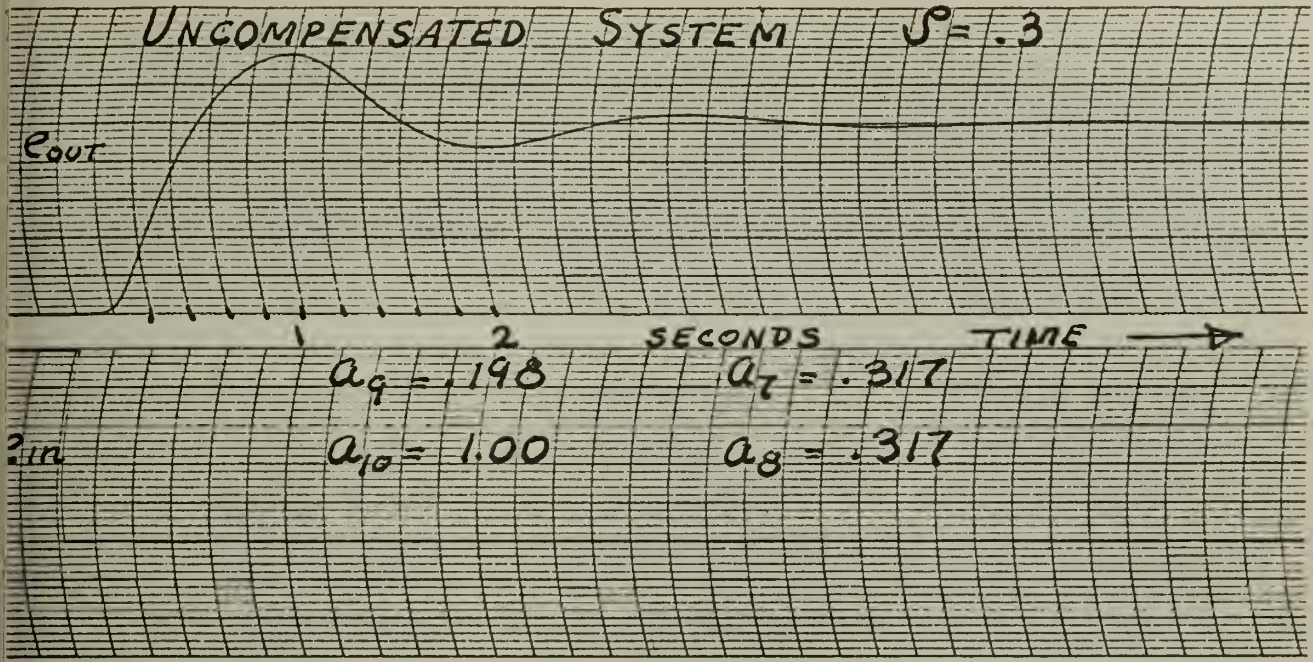
The results obtained by this type of analysis will be used again, and discussed in more detail in the next chapter in the development of a nomogram.

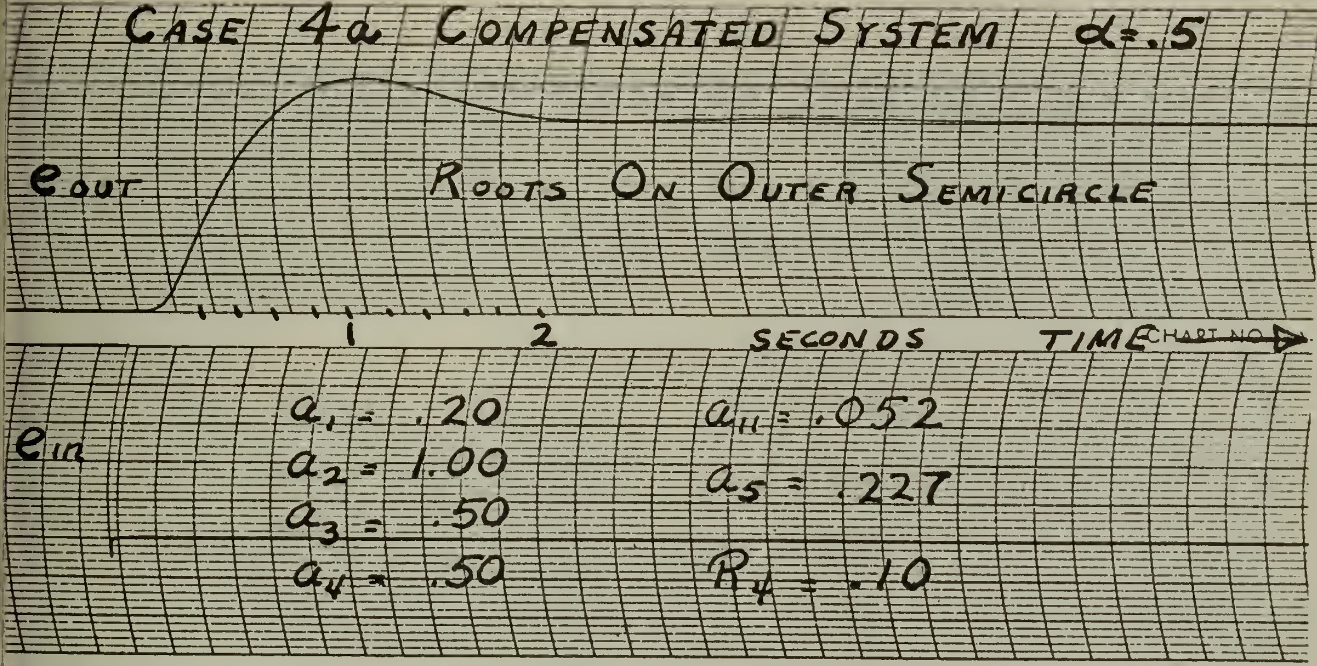
Now let's consider the transient response that can be expected from this system. If the root locus that results using this type of compensation on a second order system is studied it will be noted that if it is possible to drive the gain to very high values (which can be accomplished by variation of K_a) the complex roots can be placed on the inner circle. As a result, the real root is far out on the negative real axis. Thus we have almost a pure second order resulting with the complex roots response (having a desirable ζ) being the dominant roots without any question at all.



However if the roots are placed on the outer semi circle, there is some doubt as to whether those roots will be dominant or not. Therefore an analoque computer set up was utilized to determine the transient response of the system when the roots were placed on the outer semi circle, when the roots were real, and finally when the roots were placed on the inner circle. (See Appendix B for analoque computer set up). From the results obtained it was noted that the roots on the outer semi circle are dominant, since the real root lies very close to the compensator pole which is a zero of the system function and almost completely cancels out the effect of this real root.

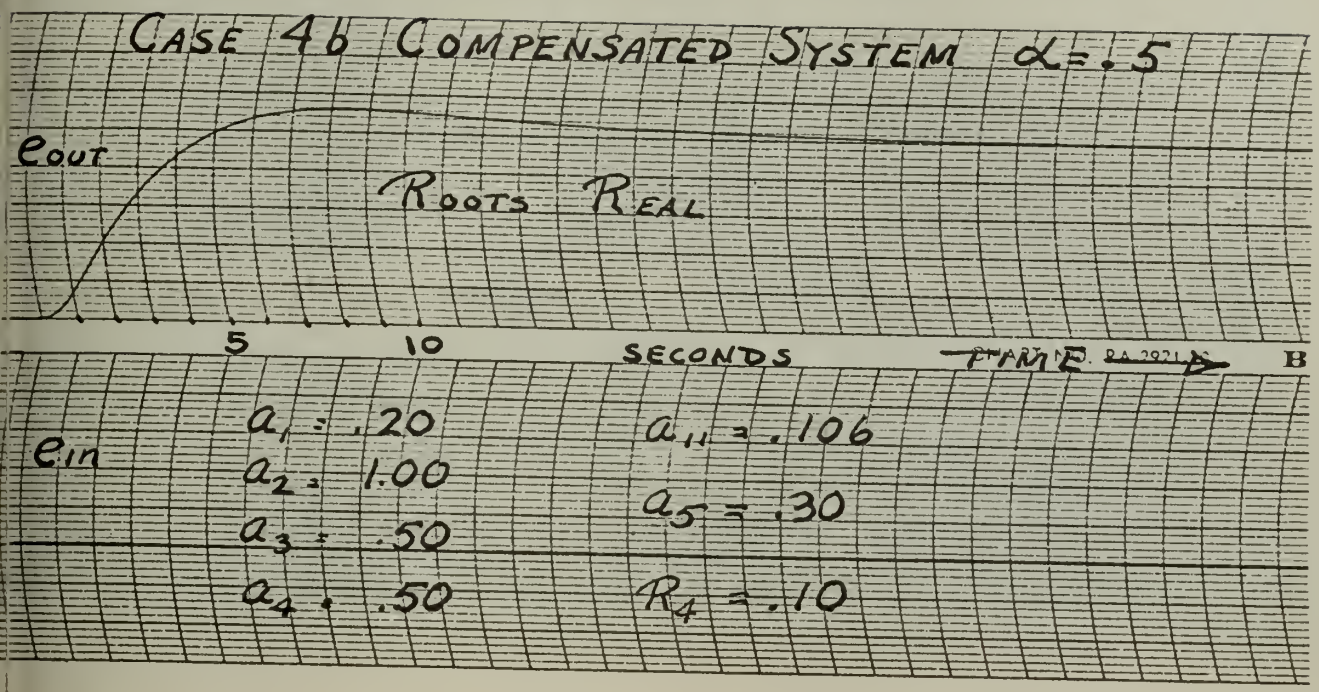
To illustrate these results the following tranient responses are presented for various compensator pole positions and various initial to prove the dominance of the roots and the variation of the root position by increasing K_a.

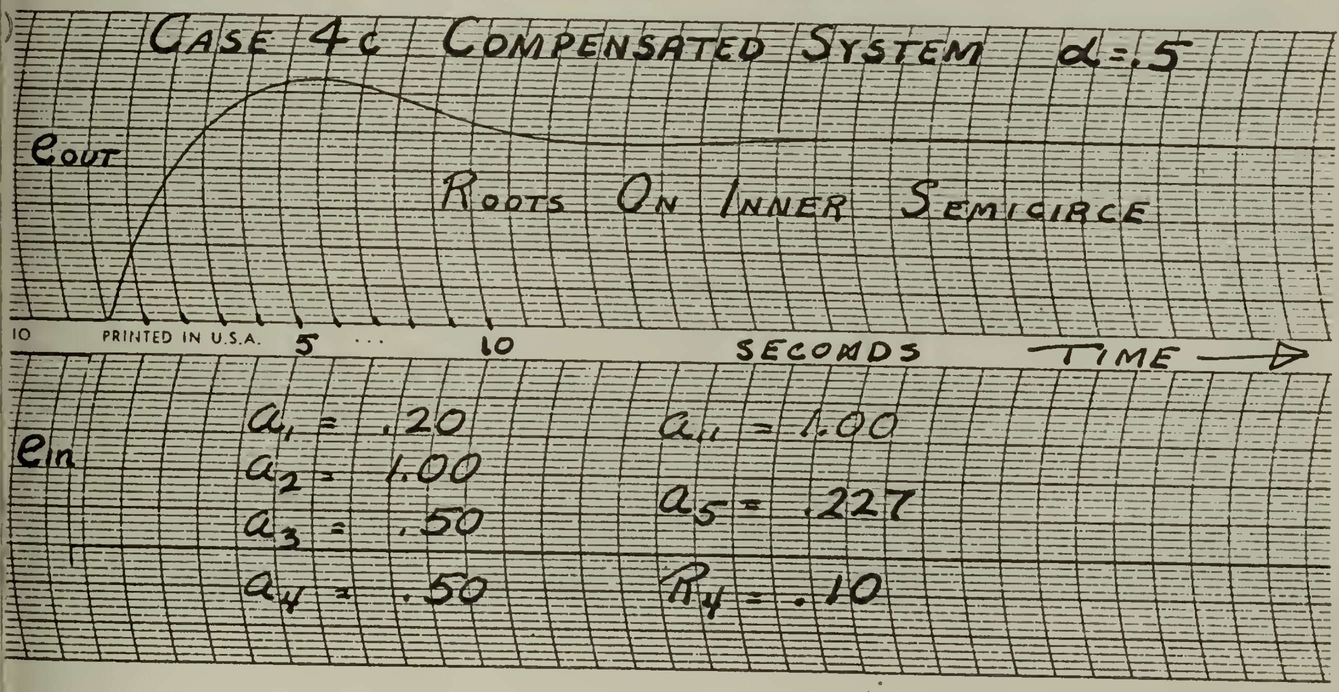


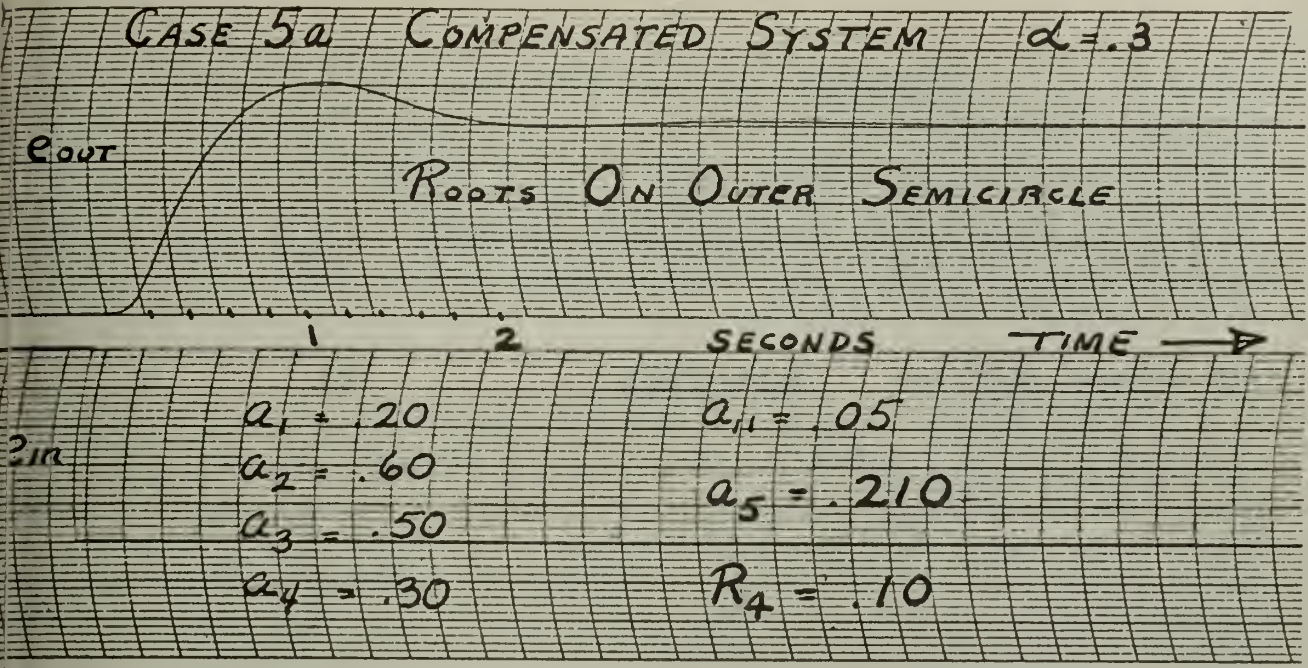


igures 2-11 a - t

LLUSTRATE THE TRANSIENT RESPONSE FOR THE INITIALLY UNCOMPENSATED SYSTEM, AND OW THE ADDITION OF APPROXIMATE ACCELERATION FEEDBACK AFFECTS IT FOR VARIOUS OSITIONS OF THE COMPENSATOR POLE.


CASE 4b COMPENSATED SYSTEM $\alpha = .5$
e_{OUT}
ROOTS REAL
5
10
SECONDS
TIME
B
e_{in}
$a_1 = .20$
$a_2 = 1.00$
$a_3 = .50$
$a_4 = .50$
$a_{11} = .106$
$a_5 = .30$
$R_4 = .10$


CASE 4c COMPENSATED SYSTEM $\alpha = .5$
e_{OUT}
ROOTS ON INNER SEMICIRCLE
10
PRINTED IN U.S.A.
5
10
SECONDS
TIME
e_{in}
$a_1 = .20$
$a_2 = 1.00$
$a_3 = .50$
$a_4 = .50$
$a_{11} = 1.00$
$a_5 = .227$
$R_4 = .10$



CASE 5a COMPENSATED SYSTEM $\alpha = .3$

$a_1 = .20$

$a_2 = .60$

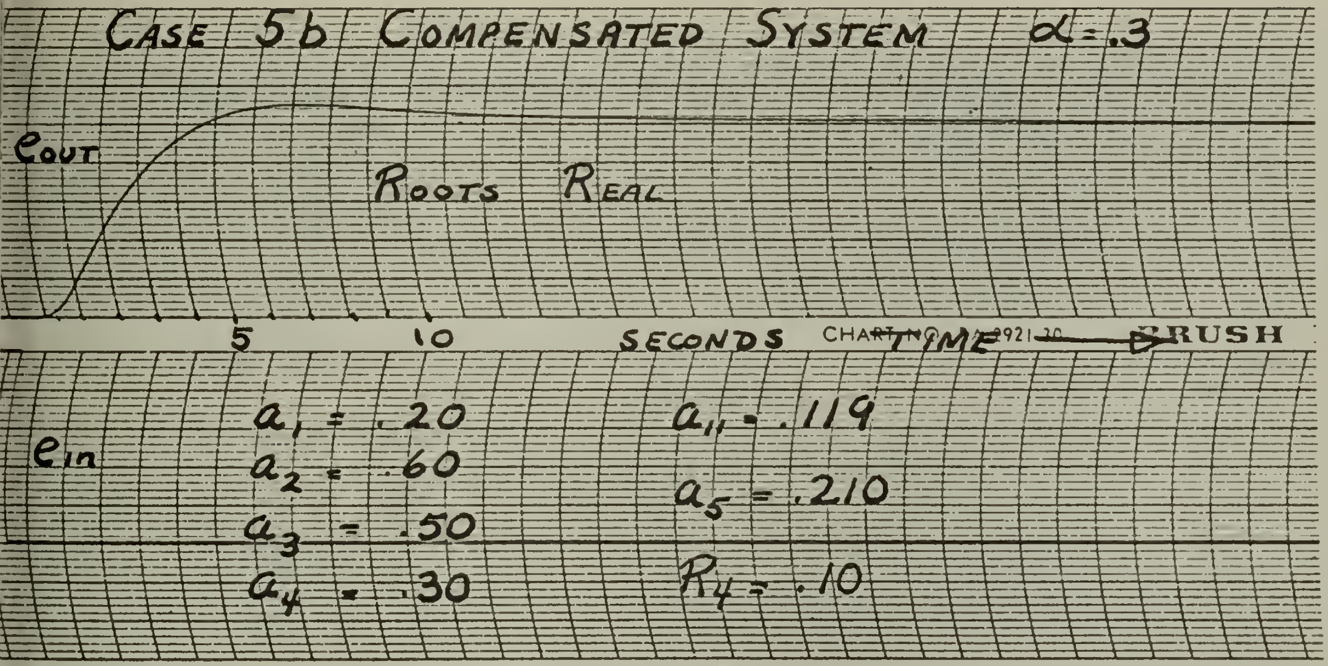
$a_3 = .50$

$a_4 = .30$

$a_{11} = .05$

$a_5 = .210$

$R_4 = .10$



CASE 5b COMPENSATED SYSTEM $\alpha = .3$

$a_1 = .20$

$a_2 = .60$

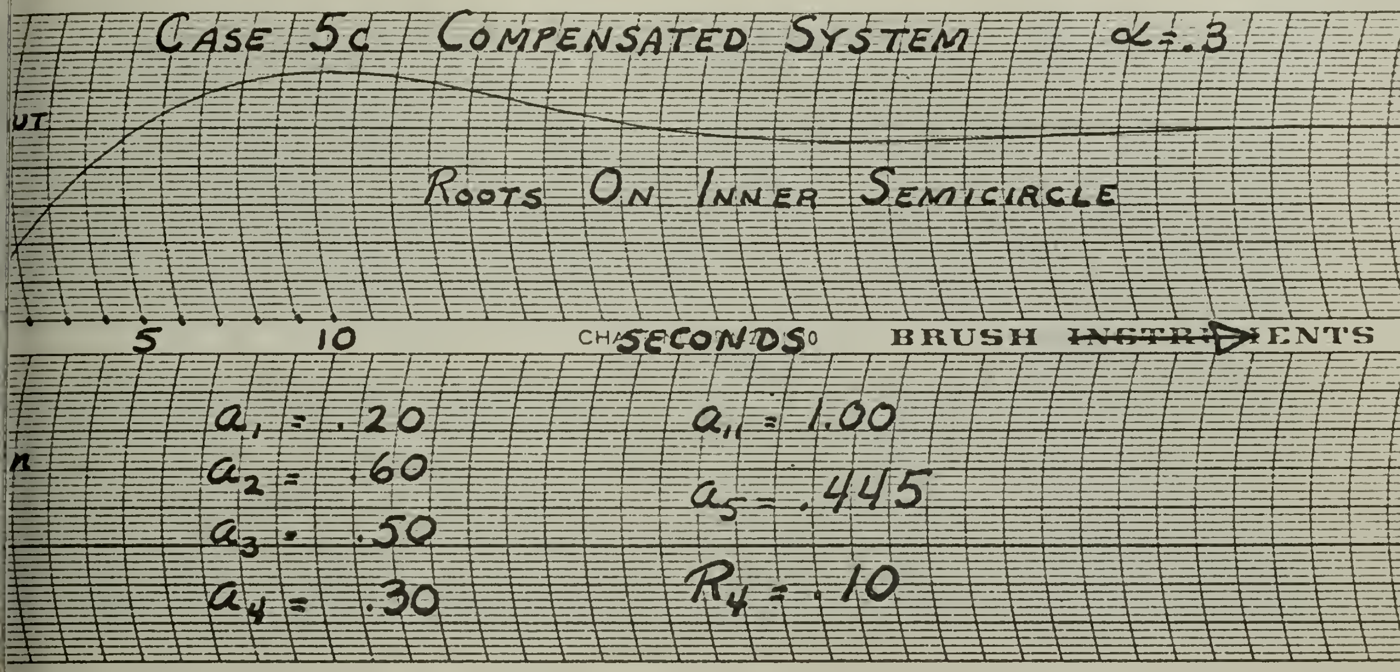
$a_3 = .50$

$a_4 = .30$

$a_{11} = .119$

$a_5 = .210$

$R_4 = .10$

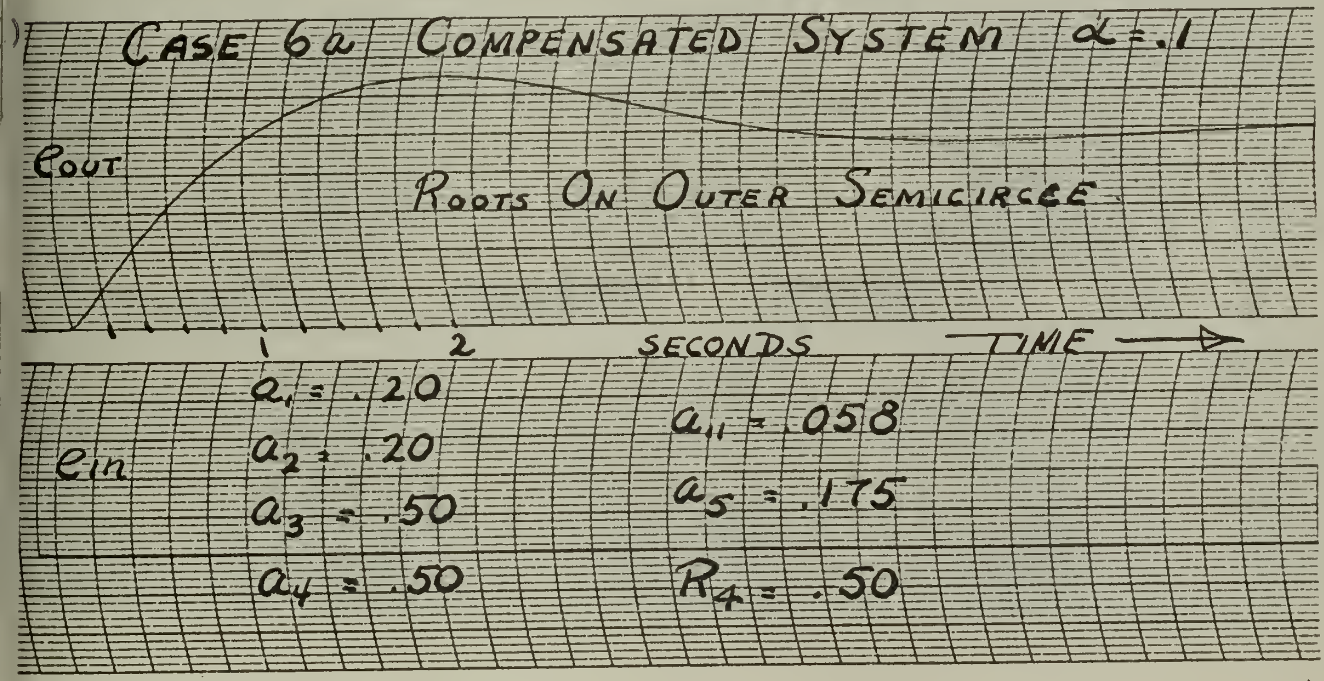


CASE 5c COMPENSATED SYSTEM $\alpha = .3$

ROOTS ON INNER SEMICIRCLE

$a_1 = .20$
$a_2 = .60$
$a_3 = .50$
$a_4 = .30$

$a_{11} = 1.00$
$a_5 = .445$
$R_4 = .10$

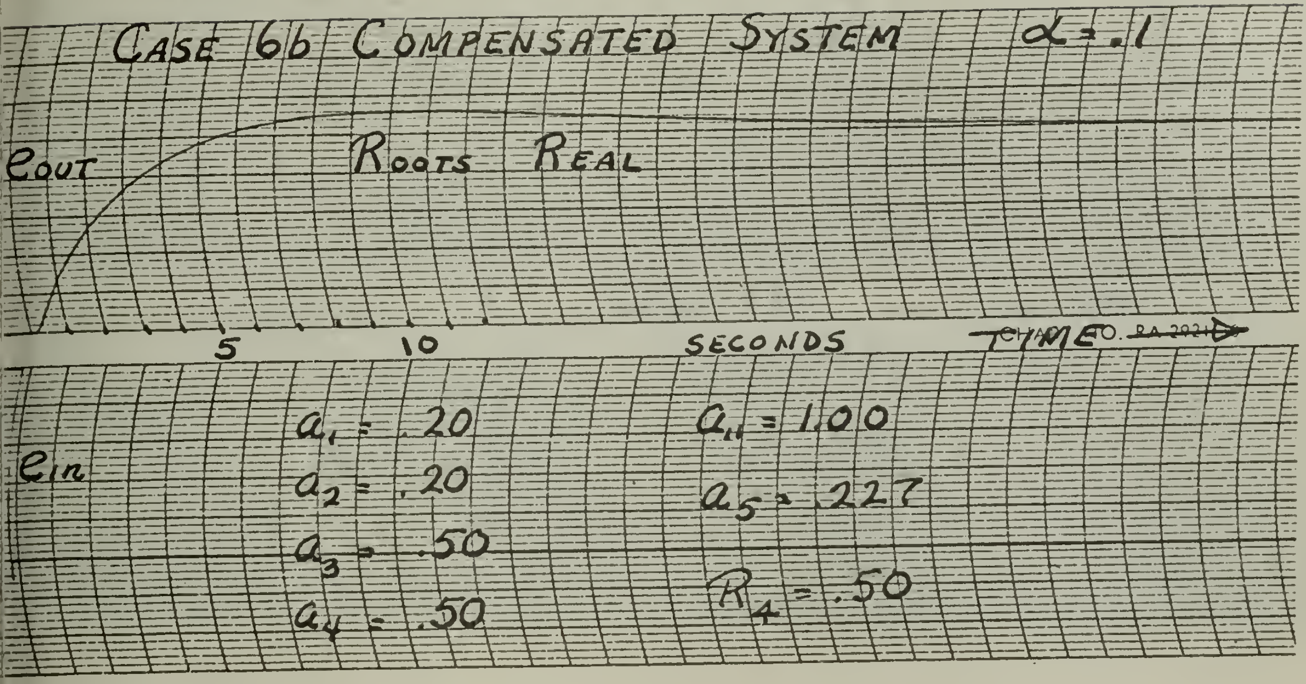


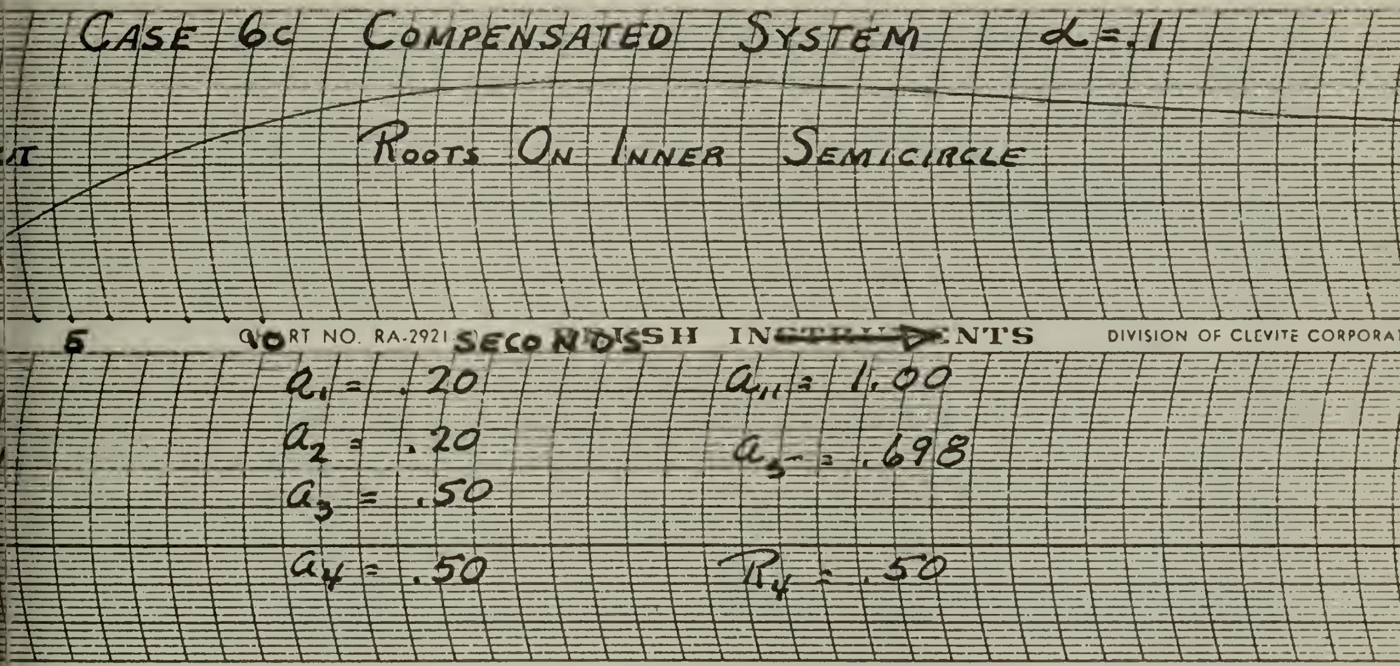
CASE 6a COMPENSATED SYSTEM $\alpha = .1$

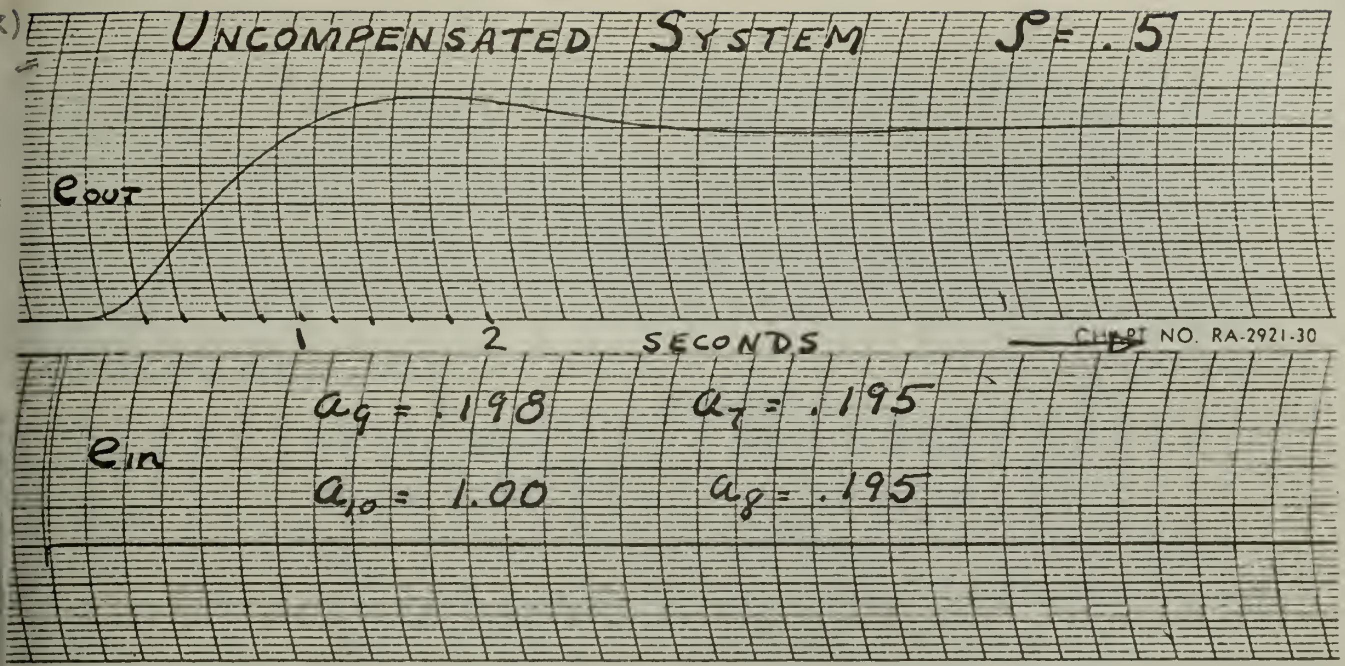
ROOTS ON OUTER SEMICIRCLE

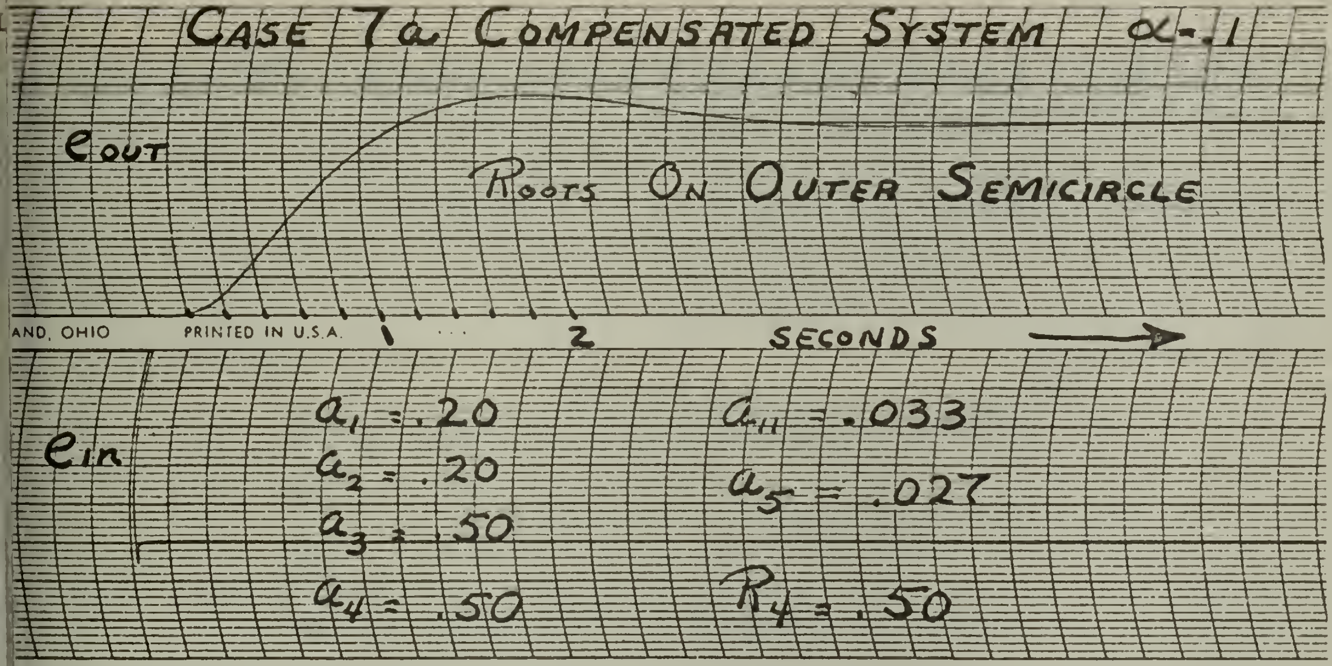
$a_1 = .20$
$a_2 = .20$
$a_3 = .50$
$a_4 = .50$

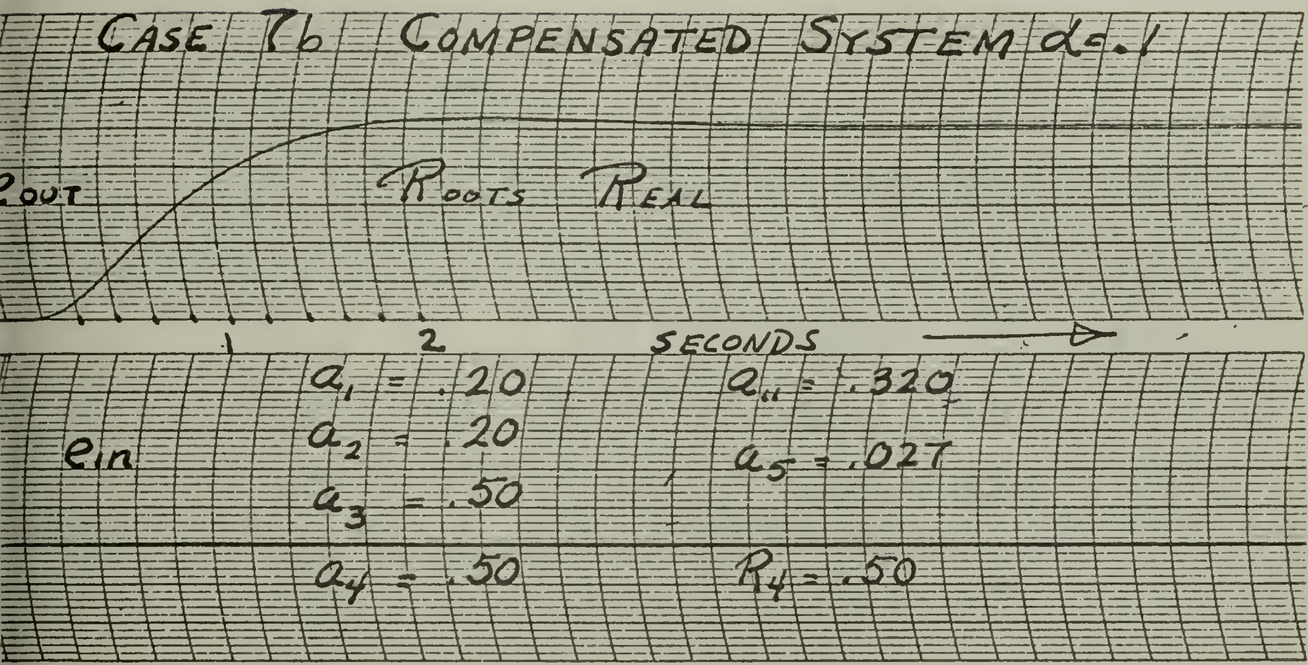
$a_{11} = .058$
$a_5 = .175$
$R_4 = .50$


CASE 6b COMPENSATED SYSTEM α = .1
ROOTS REAL
e OUT
5
10
SECONDS
TIME
e in
a₁ = .20
a₂ = .20
a₃ = .50
a₄ = .50
a₁₁ = 1.00
a₅ = .227
R₄ = .50


CASE 6c COMPENSATED SYSTEM α = .1
ROOTS ON INNER SEMICIRCLE
5
10
SECONDS
RT NO. RA-2921
BRUSH INSTRUMENTS
DIVISION OF CLEVITE CORPORA
a₁ = .20
a₂ = .20
a₃ = .50
a₄ = .50
a₁₁ = 1.00
a₅ = .698
R₄ = .50


UNCOMPENSATED SYSTEM ζ = .5
eOUT
1
2
SECONDS
CHART NO. RA-2921-30
ein
a_9 = .198
a_7 = .195
a_{10} = 1.00
a_8 = .195

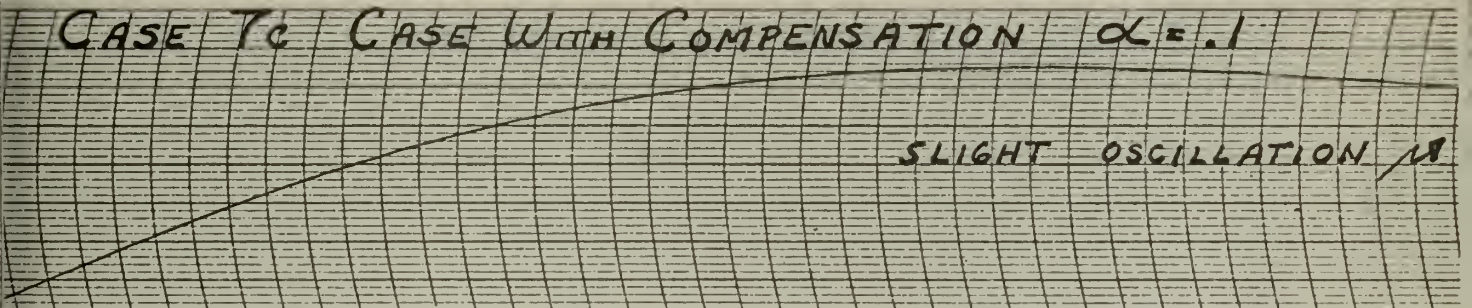

CASE 7a COMPENSATED SYSTEM α = .1
eOUT
ROOTS ON OUTER SEMICIRCLE
AND, OHIO
PRINTED IN U.S.A.
1
2
SECONDS
ein
a_1 = .20
a_2 = .20
a_3 = .50
a_4 = .50
a_{11} = .033
a_5 = .027
R_4 = .50



CASE 7b COMPENSATED SYSTEM $\alpha = .1$

ROOTS REAL

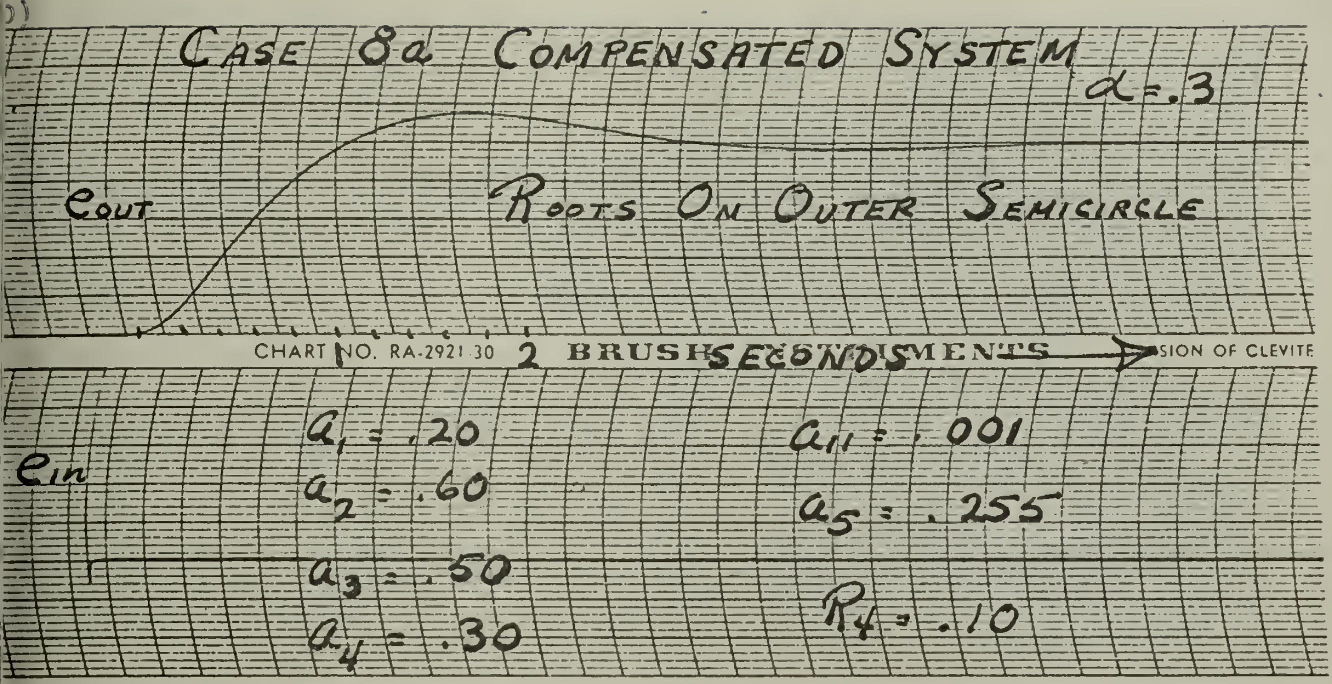
$a_1 = .20$
$a_2 = .20$
$a_3 = .50$
$a_4 = .50$
$a_{11} = .320$
$a_5 = .027$
$R_4 = .50$

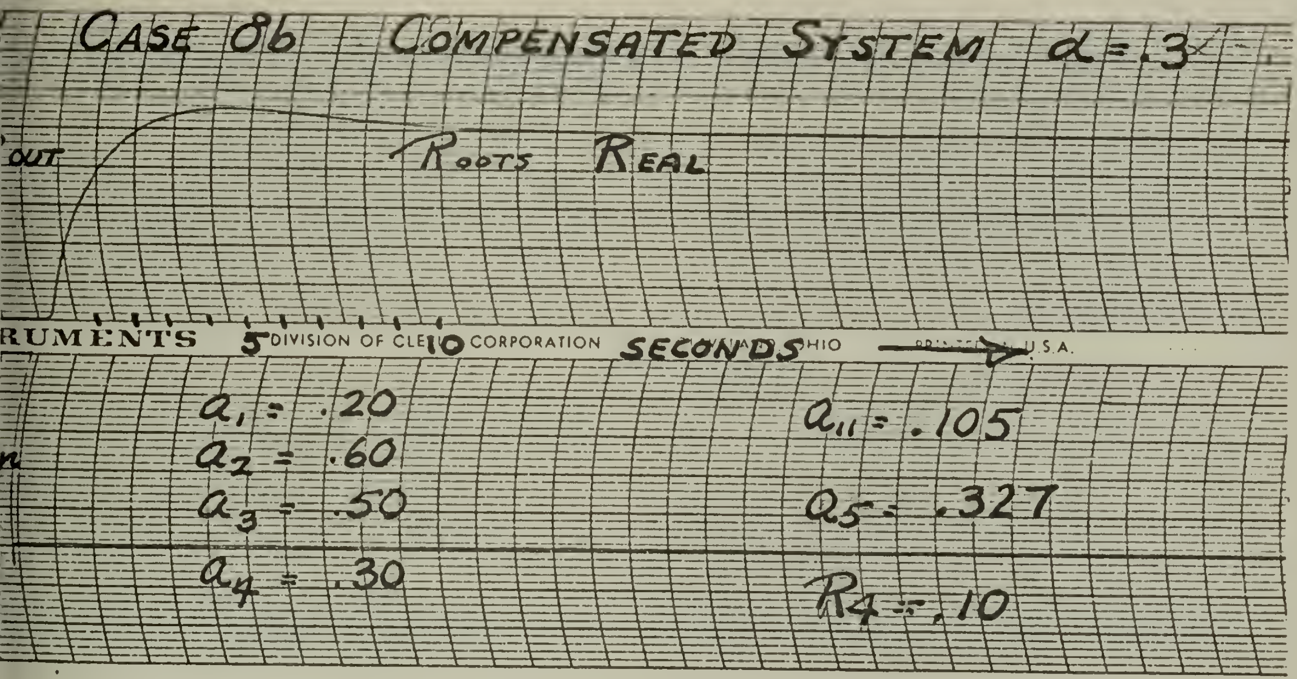


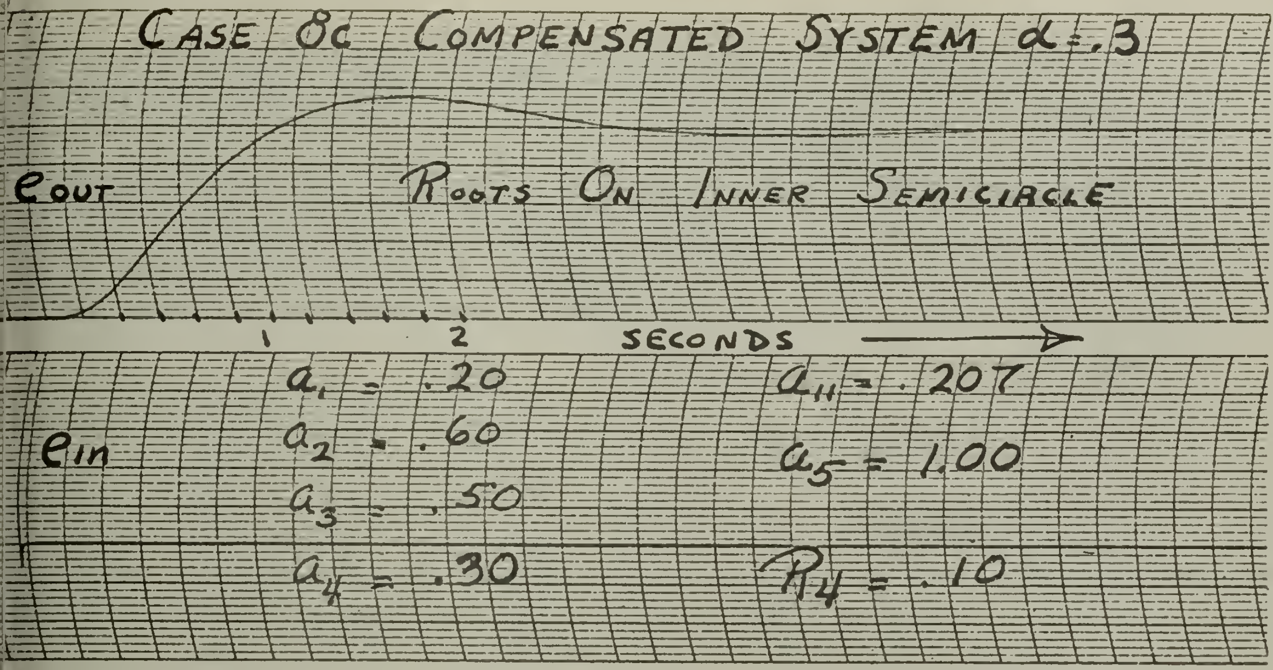
CASE 7c CASE WITH COMPENSATION $\alpha = .1$

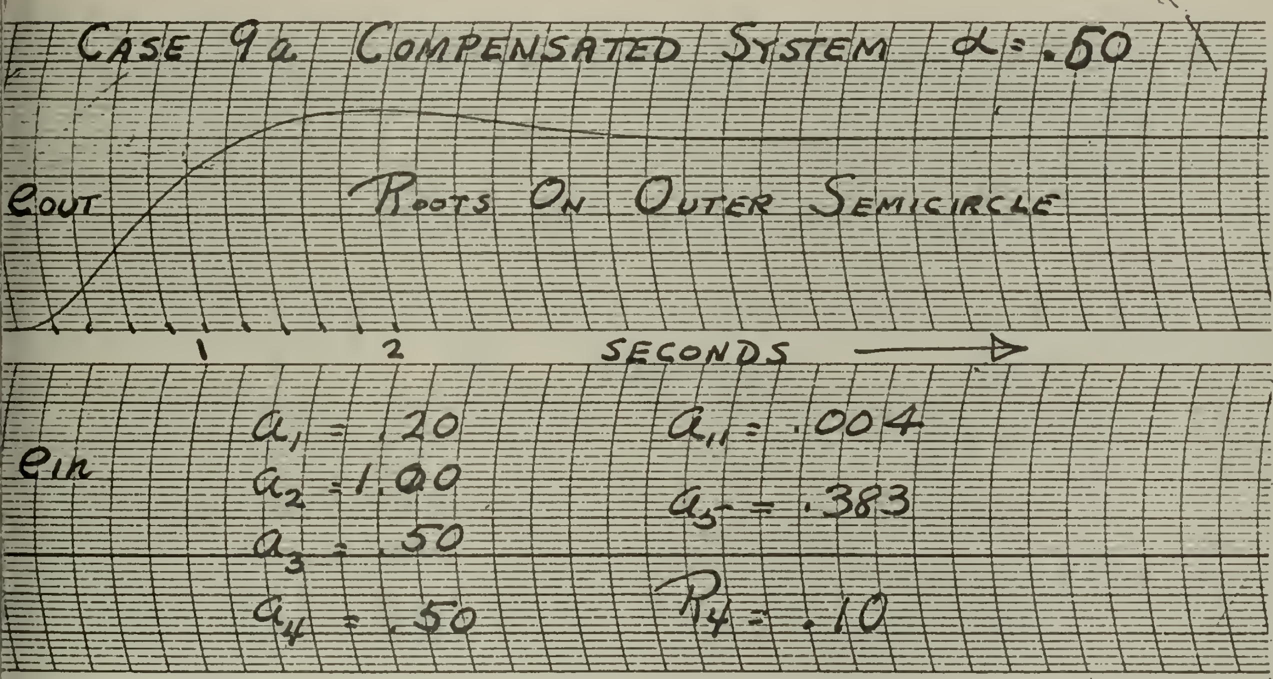
SLIGHT OSCILLATION

$a_1 = .20$
$a_2 = .20$
$a_3 = .50$
$a_4 = .50$
$a_{11} = .458$
$a_5 = 1.00$
$R_4 = .50$

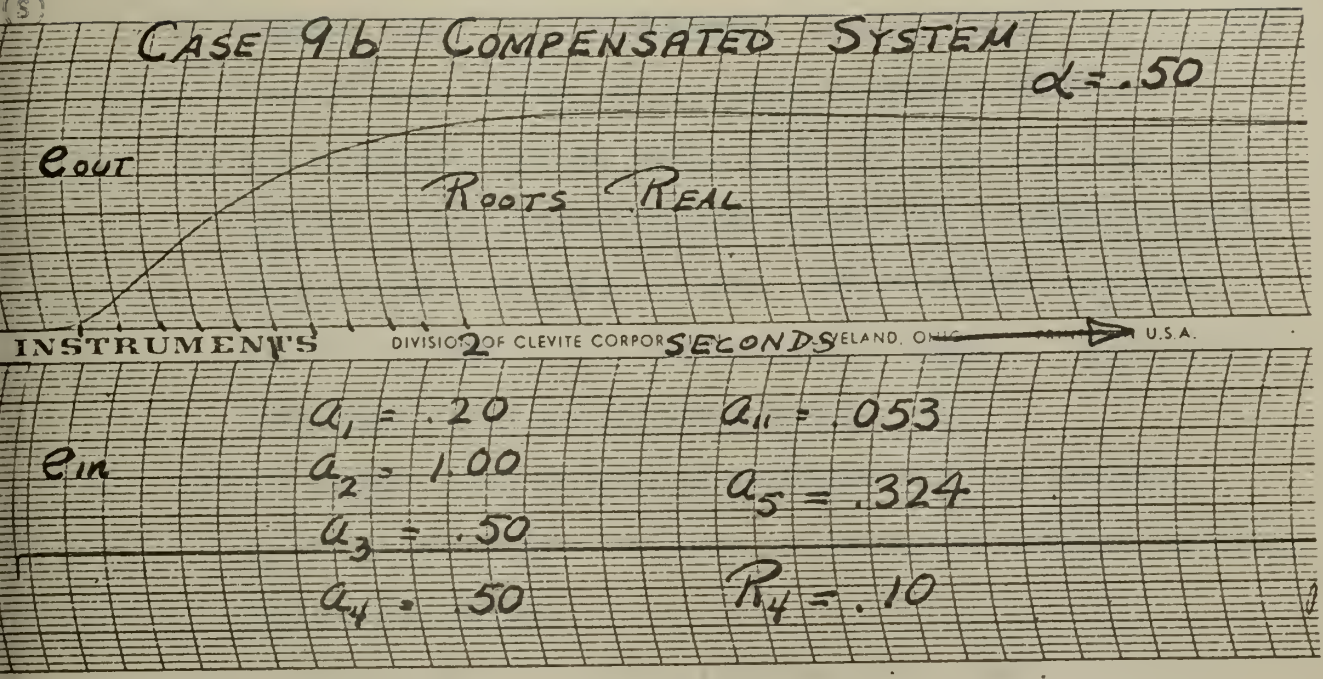

CASE 8a COMPENSATED SYSTEM
α = .3
e_OUT
ROOTS ON OUTER SEMICIRCLE
CHART NO. RA-2921-30
2
BRUSH INSTRUMENTS
SECONDS
SION OF CLEVITE
e_in
a_1 = .20
a_2 = .60
a_3 = .50
a_4 = .30
a_11 = .001
a_5 = .255
R_4 = .10


CASE 8b COMPENSATED SYSTEM α = .3
OUT
ROOTS REAL
RUMENTS
5
DIVISION OF CLE
10
CORPORATION
SECONDS
HIO
U.S.A.
n
a_1 = .20
a_2 = .60
a_3 = .50
a_4 = .30
a_11 = .105
a_5 = .327
R4 = .10

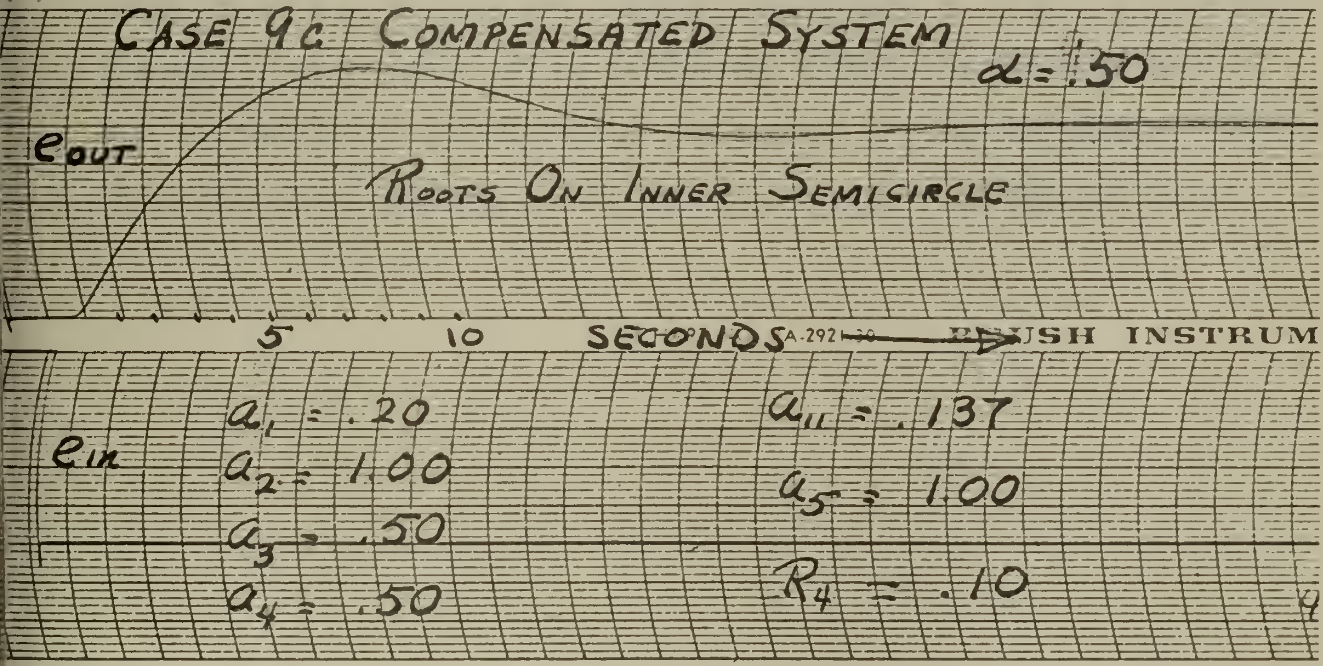

CASE 8c COMPENSATED SYSTEM α = .3


CASE 9a COMPENSATED SYSTEM α = .50

(S)


CASE 9b COMPENSATED SYSTEM
α = .50
e_OUT
ROOTS REAL
INSTRUMENTS
DIVISION OF CLEVITE CORPOR
ELAND. OH
U.S.A.
2
SECONDS
e_in
a₁ = .20
a₂ = 1.00
a₃ = .50
a₄ = .50
a₁₁ = .053
a₅ = .324
R₄ = .10

(T)


CASE 9c COMPENSATED SYSTEM
α = .50
e_OUT
ROOTS ON INNER SEMICIRCLE
5
10
SECONDS
A-292
BRUSH INSTRUM
e_in
a₁ = .20
a₂ = 1.00
a₃ = .50
a₄ = .50
a₁₁ = .137
a₅ = 1.00
R₄ = .10

To summarize the effects of employing this type of acceleration feedback on a second order system, the following statements are made:

1) For an original system, having a low ζ_i, the root locus will consist of pure circles and semi circles for a compensator pole having a value of (.05P) or less. The deviations from these circles and semi circles can be noted on pages for different compensator poles and different initial ζ_i.
2) The complex roots obtained by variation of K_a are dominant. K_a can be set to give the roots in the desired location, improving system performance.
3) If the gain K_a can be made high enough, the result will be a second order system response with roots having a desired ζ_f.

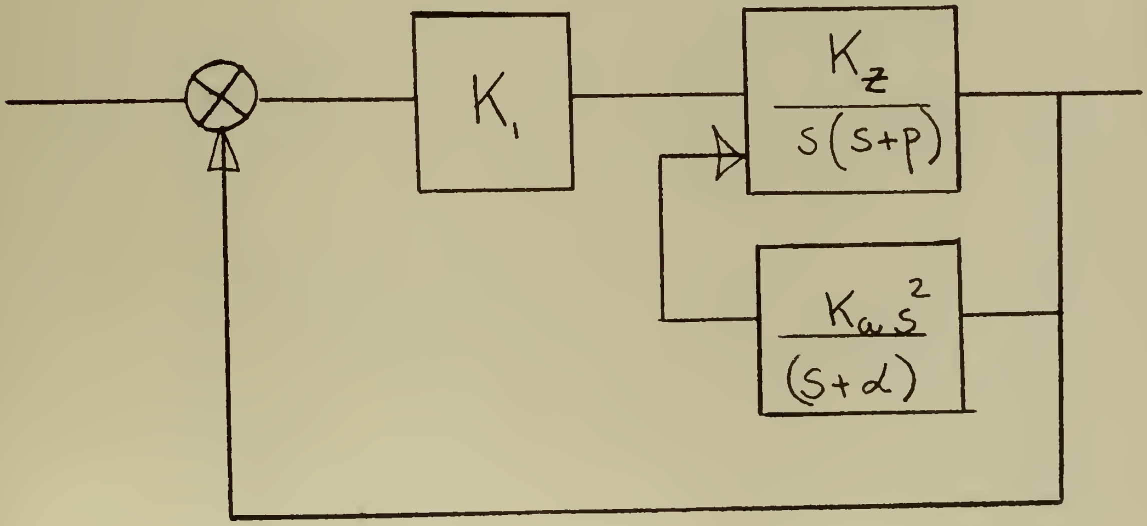
Thus it can be easily seen that an improvement in system response can be definitely obtained. This improvement can be established simply by variation of the acceleration feedback gain K_a, and if certain limitations are met, a set of rules can be established whereby the exact value of K_a and α required to yield roots in the desired location are simply obtained. This generalized approach is the topic to be covered in Chapter Three.

CHAPTER THREE

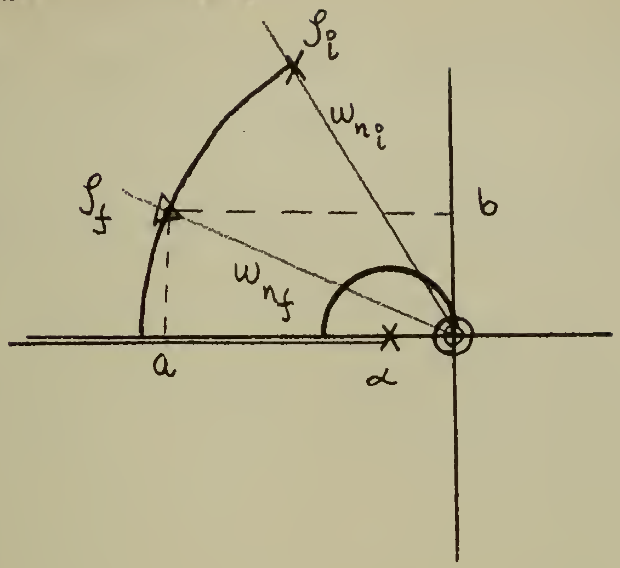
THE NOMOGRAM

Once it has been decided to use the type of compensation discussed in Chapter Two, the value of the compensator pole α and the feedback gain K_a are readily available. For the second order system, a graphical approach utilizing a nomogram to obtain the appropriate value of α and K_a will be presented. On the nomogram, the actual desired position of the roots can be selected and the required values for α and K_a can be either read off directly, or calculated quite easily. It should be pointed out that this method may also provide answers which are a very good approximation to the actual values for a third order system which has its real root far out on the negative real axis.

In the following chapter this method shall be illustrated, and several example problems will be worked out to stress the usefulness and importance of such a technique. Consider as previously discussed the following system



for which the root locus is:



Upon this root locus, it can be noted that ζ_i and ω_{n_i} define the location of the roots of the system before compensation. After compensation, it is desired perhaps to have the roots located at $s = -a + jb$ which is defined by ζ_f and ω_{n_f} . The following indentities should be kept in mind

$$a^2 + b^2 = \omega_{n_f}^2$$

$$\cos \theta = \frac{a}{\omega_{n_f}} = \zeta_f$$

$$K_1 K_2 = \omega_{n_i}^2$$

Since the root locus is a means of obtaining the roots of the characteristic equation, lets solve the characteristic equation for K_a and notice the results. That is:

$$s^3 + (K_a K_z + p + \alpha) s^2 + (K_1 K_z + p\alpha) s + K_1 K_z \alpha = 0$$

$$K_a K_z = - \frac{s^3 - (p+\alpha)s^2 - (K_1 K_z + p\alpha)s - K_1 K_z \alpha}{s^2}$$

If the final position of the desired roots is $s = - a + jb$, and

$$K_a K_z = - s - (p+\alpha) - \frac{(K_1 K_z + p\alpha)}{s} + \frac{K_1 K_z \alpha}{-s^2}$$

Then

$$K_a K_z = -(-a+jb) - (p+\alpha) - \frac{(K_1 K_z + p\alpha)(-a+jb)}{(-a+jb)(-a+jb)} - \frac{K_1 K_z \alpha (a^2 - b^2 + j2ab)}{(a^2 - b^2 + j2ab)(a^2 - b^2 - j2ab)}$$

Therefore, separating real and imaginary components

$$K_a K_z = \left[+a - (p+\alpha) + \frac{(K_1 K_z + p\alpha)}{a^2+b^2} - \frac{K_1 K_z \alpha (a^2-b^2)}{(a^2+b^2)^2} \right]$$

$$+ j\left(-b + \frac{(K_1 K_z + p\alpha) b}{a^2+b^2} - \frac{2ab K_1 K_z \alpha}{(a^2+b^2)^2} \right)$$

However, since K_a is a gain quantity and must be a real number, the imaginary component must be zero and the only part of importance is the real term. Thus, solving for K_a in terms of the real part yields

$$K_a K_z = +a - (p+\alpha) + \frac{(K_1 K_z + p\alpha) a}{a^2+b^2} - \frac{K_1 K_z \alpha (a^2-b^2)}{(a^2+b^2)^2}$$

SINCE $(a^2+b^2)^2 = \omega_{n_f}^4$

$$K_a K_z = \frac{a\omega_{n_f}^4 - (p+\alpha)\omega_{n_f}^4 + (K_1 K_z + p\alpha)\, a\omega_{n_f}^2}{\omega_{n_f}^4}$$

$$+ \frac{K_1 K_z \alpha\left[-(a^2-b^2)\right]}{\omega_{n_f}^4}$$

LET $p = 2\zeta_i \omega_{n_i}$, $\alpha = \zeta_i \omega_{n_i} X$ $\quad X = .1, .2,$ etc

$$K_a K_z = \frac{\zeta_f \omega_{n_f}^5}{\omega_{n_f}^4} \; \frac{-\omega_{n_f}^4(2\zeta_i \omega_{n_i} + \zeta_i \omega_{n_i} X)}{\omega_{n_f}^4}$$

$$+ \frac{(\omega_{n_i}^2 + 2\zeta_i \omega_{n_i}^2 X)\,\zeta_f \omega_{n_f}^3}{\omega_{n_f}^4}$$

$$+ \frac{\omega_{n_i}^3 \zeta_i X (a^2-b^2)}{\omega_{n_f}^4}$$

but the last term

$$\frac{\omega_{n_i}^{3}\,\zeta_i X\left(a^2-b^2\right)}{\omega_{n_t}^{4}} = \frac{\zeta_i\,\omega_{n_i}^{3} X\left[\zeta_t^{2}\,\omega_{n_t}^{2}-\sin^{2}\theta\,\omega_{n_t}^{2}\right]}{\omega_{n_t}^{4}}$$

$$= \frac{\zeta_i X\,\omega_{n_i}^{3}\,\omega_{n_t}^{2}\left[\zeta_t^{2}-\left(1-\zeta_t^{2}\right)\right]}{\omega_{n_t}^{4}}$$

$$= \frac{\zeta_i \bar{X}\,\omega_{n_i}^{3}\left[2\zeta_t^{2}-1\right]}{\omega_{n_t}^{2}}$$

Therefore

$$K_a K_z = \zeta_t\,\omega_{n_t} - 2\zeta_i\,\omega_{n_i} - 2\zeta_i\,\omega_{ni} X + \frac{\omega_{n_i}^{2}\,\zeta_t\left[1+2\zeta_i^{2} X\right]}{\omega_{n_t}^{2}} - \frac{\zeta_i X\,\omega_{n_i}^{3}\left[2\zeta_t^{2}-1\right]}{\omega_{n_t}^{2}}$$

$$K_a K_z = a - (p+\alpha) + \frac{a\omega_{n_i}^2\left[1 + 2\zeta_i^2 x\right]}{\omega_{n_t}^2} - \frac{\alpha K_1 K_z\left(2\zeta_t^2 - 1\right)}{\omega_{n_t}^2}$$

$$= a - (p+\alpha) + \frac{\omega_{n_i}^2}{\omega_{n_t}^2}\left[a + 2a\zeta_i^2 x - 2\alpha\zeta_t^2 + \alpha\right]$$

$$* \; K_a K_z = a - (p+\alpha) + \frac{K_1 K_z}{\omega_{n_t}^2}\left[a\left(1 + 2\zeta_i^2 x\right) + \alpha\left(1 - 2\zeta_t^2\right)\right]$$

The above* is the general expression regardless where the desired roots may be, (i.e., the roots in this case can be referred to the outer or the inner semi circle).

By various manipulations, it can also be expressed as

$$K_a K_z = \frac{\alpha\omega_{n_i}^2}{\omega_{n_t}^2} + \frac{a\omega_{n_i}^2}{\omega_{n_t}^2} + \frac{ap\alpha}{\omega_{n_t}^2} - (p+\alpha) + a$$

Now consider roots that be just on the outer semi circle.

In this case

$$K_1 K_z = \omega_{n_i}^2 = \omega_{n_t}^2$$

then

$$K_a K_z = a - (p + \alpha) + a(1 + 2\zeta_i^2 x) + \alpha(1 - 2\zeta_t^2)$$

$$= 2a - p + 2\zeta_i^2 x + 2\zeta_t^2$$

If ζ_i is initially small then

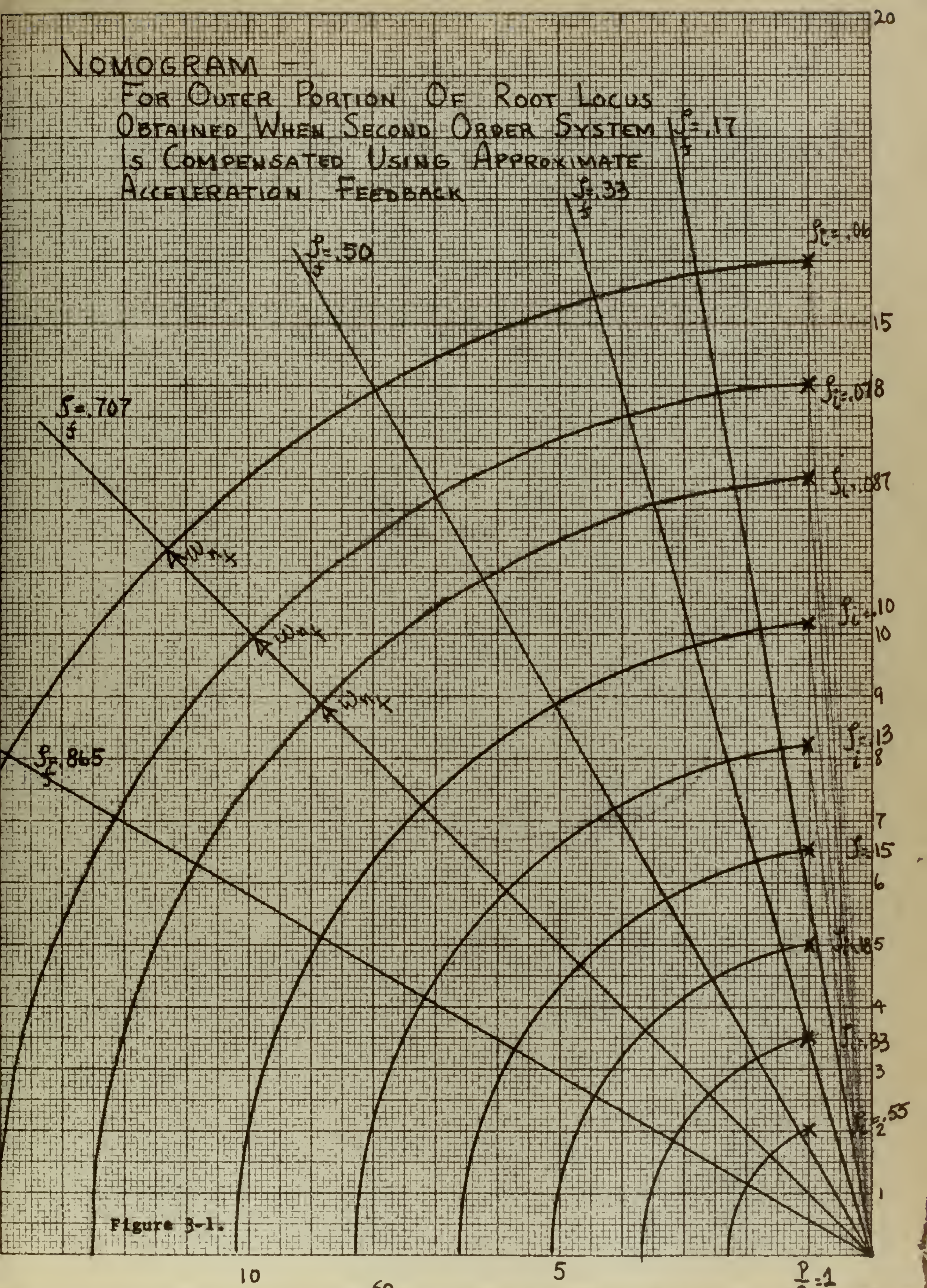
$$\boxed{K_a K_z = 2a - p - 2\zeta_t^2}$$

To summarize the results of the preceding derivations, it can be noted that if the initial positions of the roots of the system are known having ζ_i° and $\omega_{n_i}^\circ$ (These quantities are determined from the open loop gain $K_1 K_z$ and the motor pole P, $K_1 K_z$ being equal to $\omega_{n_i}^2$ while p is equal to $2\zeta_i \omega_{n_i}$), and the desired position of the roots after compensation are known (ζ_t, ω_{n_t}, obtained from specifications) the gain in the feedback loop required can be found for each compensator pole used. It should be noted that for the best results, the compensator pole should be kept close to origin as possible, (the preferable value being $\alpha = \frac{(.1)P}{2}$).

Realizing that this is true, a procedure can be set up utilizing a nomogram whereby the necessary information can be selected very readily, and the essential gain easily calculated using the equations previously derived.

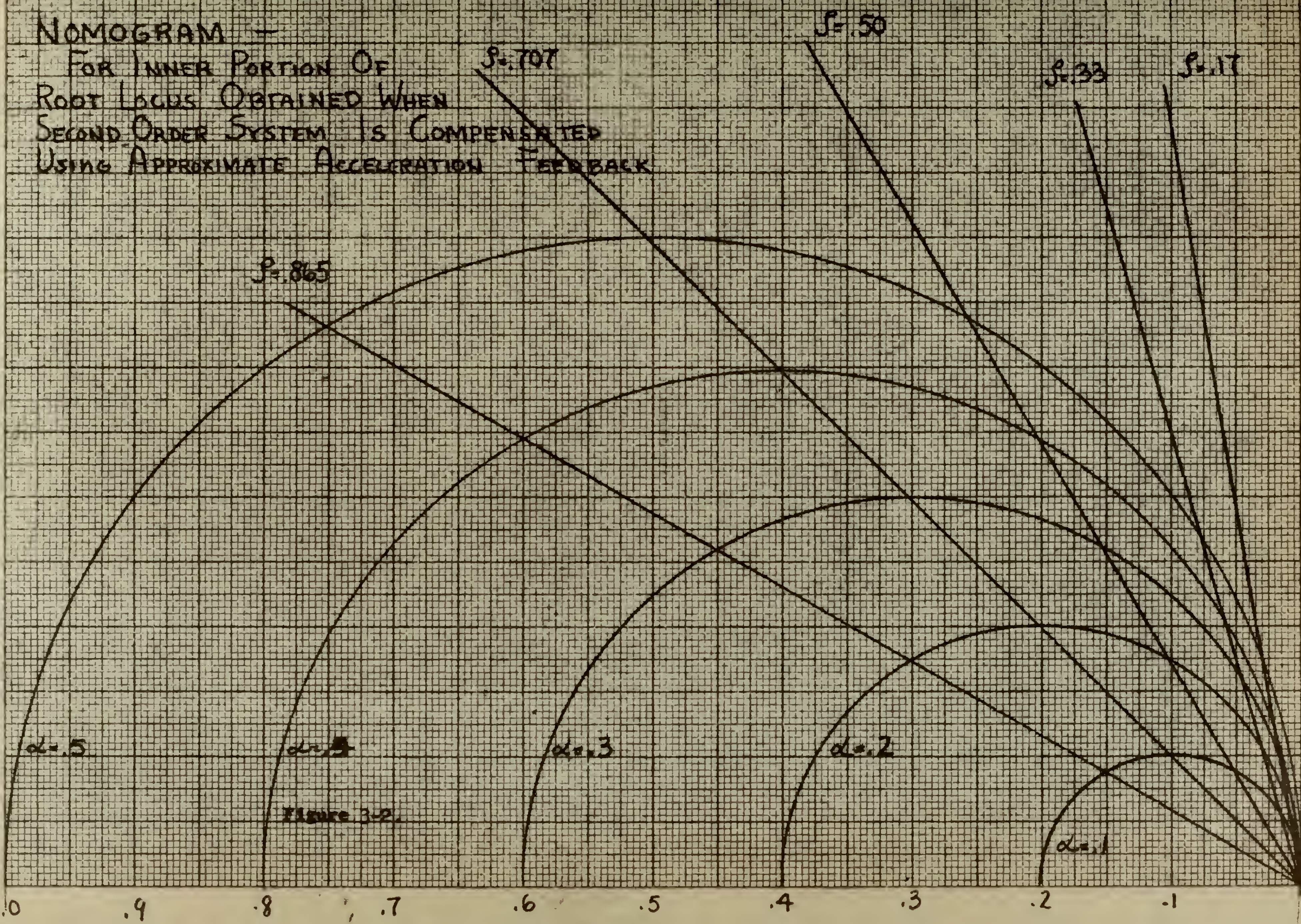
Consider the following two nomograms which approximates the roots locus as pure circles and semi circles. One is for the outer semi circle having the origin as center and radius equal to $\omega_{n_i}^\circ$.

The other is for the inner circles which have the compensator pole as center and radius equal to the magnitude of the compensator pole. For derivation or proof of these facts see Appendix A.



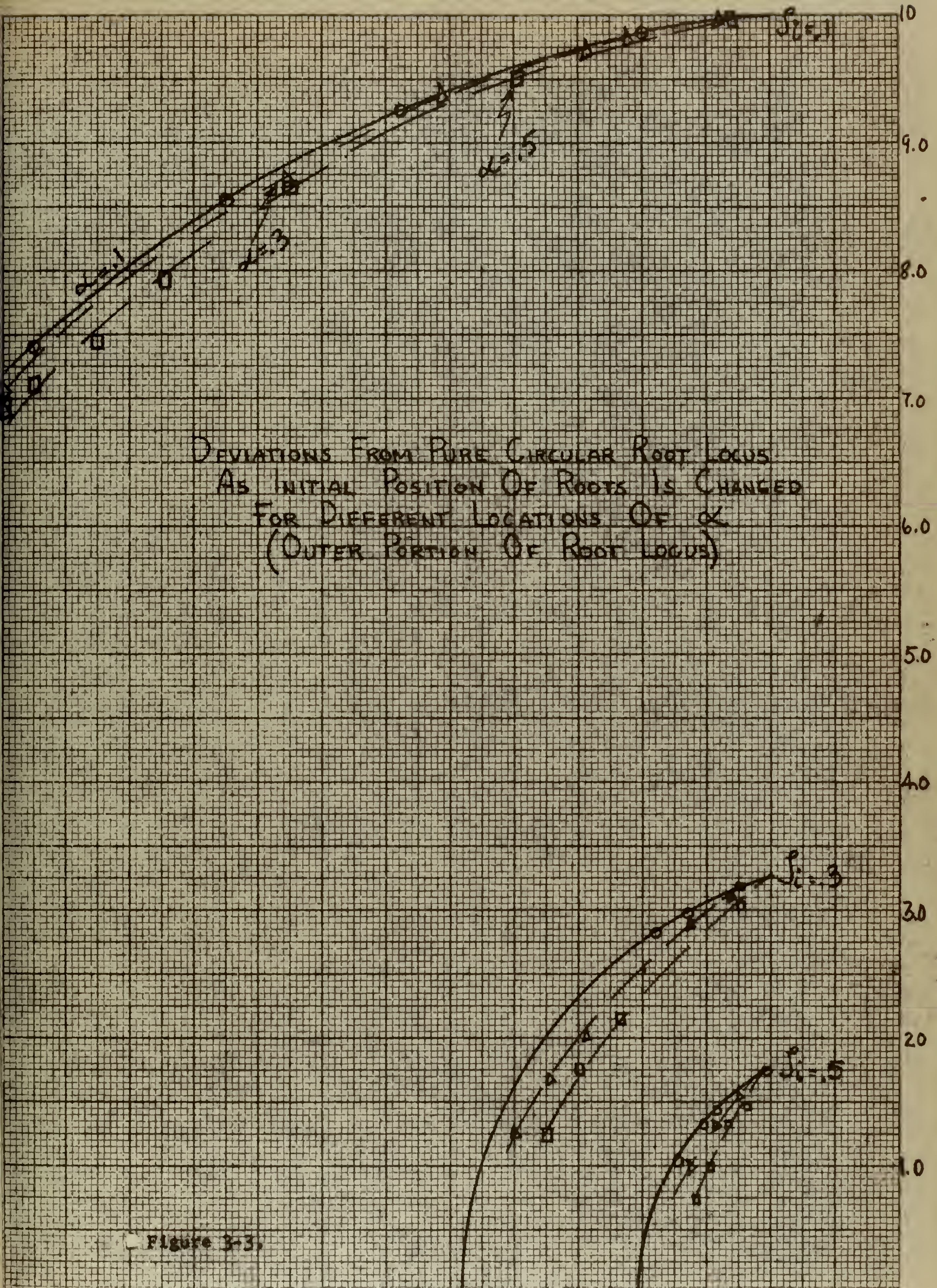
NOMOGRAM —
FOR OUTER PORTION OF ROOT LOCUS
OBTAINED WHEN SECOND ORDER SYSTEM
IS COMPENSATED USING APPROXIMATE
ACCELERATION FEEDBACK

Figure 3-1.



NOMOGRAM –
FOR INNER PORTION OF
ROOT LOCUS OBTAINED WHEN
SECOND ORDER SYSTEM IS COMPENSATED
USING APPROXIMATE ACCELERATION FEEDBACK

Figure 3-2.



DEVIATIONS FROM PURE CIRCULAR ROOT LOCUS
AS INITIAL POSITION OF ROOTS IS CHANGED
FOR DIFFERENT LOCATIONS OF α
(OUTER PORTION OF ROOT LOCUS)

Figure 3-3.

DEVIATIONS FROM PURE CIRCULAR ROOT LOCUS
AS INITIAL POSITION OF ROOTS IS CHANGED
FOR DIFFERENT LOCATIONS OF COMPENSATOR POLE α
(INNER PORTION OF ROOT LOCUS)

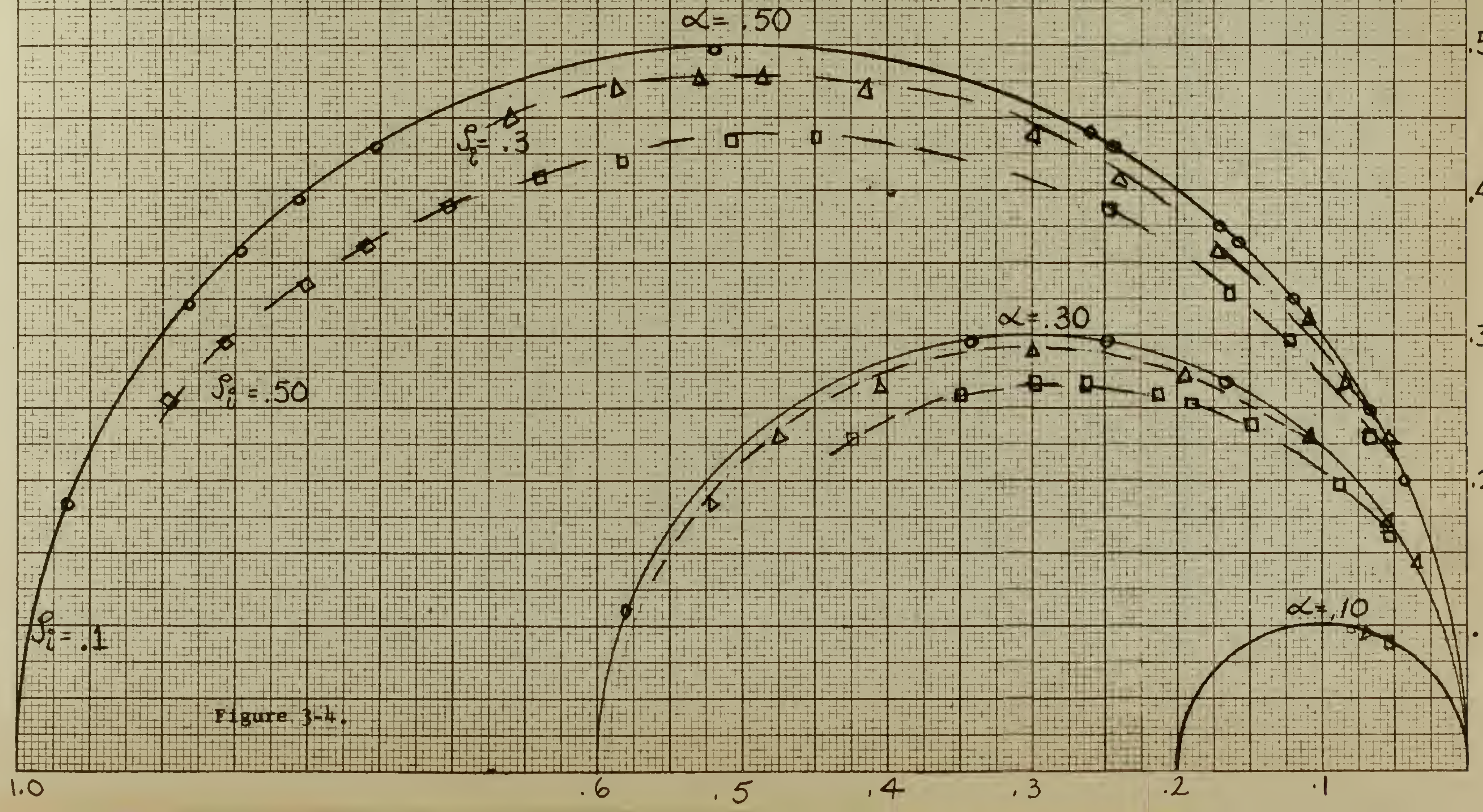


Figure 3-4.

Having these nomograms, the following procedure outlines what must be done, to obtain value of required (K_a) gain in feedback loop to have roots in the desired location.

Procedure

1) Make scale correspond to your specific problem by changing value of P and α so that

$$P_{GRAPH} = P_{ACTUAL} \qquad \alpha = \frac{P_{ACTUAL}\ X}{2}$$

where $X = .1, .2, .3,$ etc.

2) Note ζ_i, ω_{ni}, K_1 K_Z of initial system before compensation.

3) Select ζ_f of the new roots.

4) Select α which is desired.

Note that for best results X should be equal to .1.

5) Read off values of a, ζ_f, ω_{nf}

6) a) Utilizing the general expression find K_a required.

b) Or if the roots are to be on the outer semi circle use

$$K_a K_z = 2a - P - 2\zeta_f^2$$

Note also that as X increases from .1, there will actually be some deviation from the pure circles. These deviations were illustrated on the preceding pages.

Consider the following examples

a) Let $K_z = 100 \qquad K_1 = 1.0 \qquad \& \ \zeta_i = .1$

It is desired to have roots at $\zeta_f = .5$ when the motor pole

1) Since $P = 2$ it coincides identically with P on graph and

$$\alpha = \frac{2}{2}(.1) = .1$$

2) $\zeta_i = .1$ $\omega_{n_i} = 10$, since $K_1 K_Z = 100$

3) $\zeta_f = .5$ therefore if the roots are selected on the outer semi circle;

$$a = (.5)(10) = 5 \qquad \omega_{n_f} = 10$$

4) Therefore

$$K_a K_Z = 2a - p - 2\zeta_f^2$$

$$= 10 - 2 - .5 = 7.5$$

$$K_a = .075$$

Thus a gain of .075 is necessary to have roots at $\zeta = .5$ with the compensator pole initially at .1.

For the same system with the compensator pole at .3, the same value of gain is required, i.e., $K_a K_Z = 7.5$ using the approximation formula.

Using the general formula for

$$K_a K_Z = \frac{\alpha \omega_{n_i}^2}{\omega_{n_f}^2} + \frac{a \omega_{n_i}^2}{\omega_{n_f}^2} + \frac{a p \alpha}{\omega_{n_f}^2} - (p + \alpha) + a$$

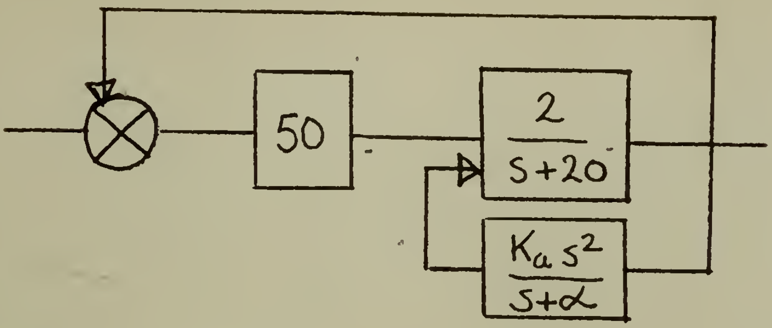
$$= 8.3$$

whereas previously if the general formula had been used for $\alpha = .1$

$$K_a K_Z = 8.1$$

Therefore it can be seen that the accurate answer can always be obtained with the general expression whereas the approximation formula yields just an approximation.

For the system shown below find the gain required to have roots at $\zeta_f = .6$, and a fast response time.



1) Since $P_{actual} = 100$

$P_{graph} = 100$

2) $\rho_i = .3$, $\omega_{n_i} = 31.9$ $K_1 K_2 = 100$

3) $\rho_f = .6$ therefore if the roots are selected on outer semi circle where they should be for a fast response time.

$$a = (.6)(31.9) = 19.14 \qquad \omega_{n_f} = 31.9$$

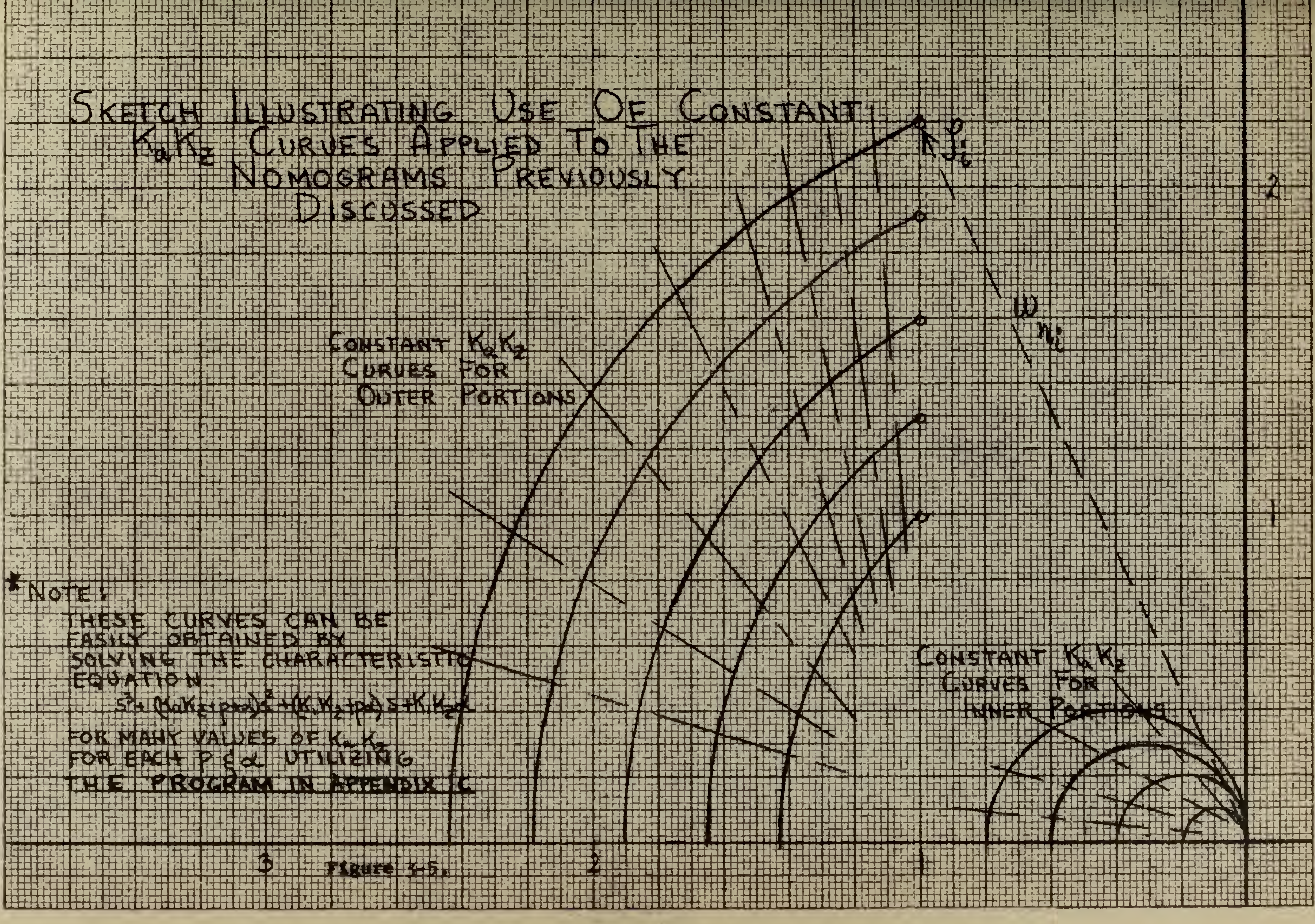
4) $$K_a K_z = 2a - p - 2\rho_f^2$$

$$= 38 - 20 - .72 = 17.3$$

5) $$K_a = \frac{17.3}{2} = 8.65$$

Instead of having to read the values of a, ρ_f & ω_{n_f} off the graph and using the equations already desired, a set of constant $K_a K_Z$ curves can be plotted on the nomogram so that the value of $K_a K_Z$ and thus K_a can be read off the curve directly. To give the reader an idea of what is implied here, consider the following nomogram with constant $K_a K_Z$ lines. Once this curve is obtained, one need only know the initial damping ratio ρ_i and natural frequency ω_{n_i}, select the appropriate α for the compensator pole, and the location of the roots in order to find the value of K_a required directly from the curves.

SKETCH ILLUSTRATING USE OF CONSTANT K_aK_2 CURVES APPLIED TO THE NOMOGRAMS PREVIOUSLY DISCUSSED


CONSTANT K_aK_2 CURVES FOR OUTER PORTIONS
CONSTANT K_aK_2 CURVES FOR INNER PORTIONS
ρ_i
ω_{n_i}
2
1
3
2
1

*NOTE:

THESE CURVES CAN BE EASILY OBTAINED BY SOLVING THE CHARACTERISTIC EQUATION

$$s^3+(K_aK_2+p\alpha)s^2+(K_1K_2+p\alpha)s+K_1K_2\alpha$$

FOR MANY VALUES OF K_aK_2 FOR EACH P & α UTILIZING THE PROGRAM IN APPENDIX C

Figure 3-5.

So far the third root which is real has been completely eliminated from the discussion. However, its presence is important and must be taken into account in explaining the behavior of the system.

When the acceleration feedback gain is relatively low (complex roots on outer semi circle) the real root will be very close to the compensator pole α which happens to be a zero of the overall system function.

Thus the effects of this root are almost completely cancelled out. When the gain is increased to such high values so as to have complex roots on the inner semi circle, the real root be very far out on the negative real axis. As a result, its effect will damp out rather rapidly. However, in spite of the cancellation in many respects of its impact on the system, it is still desirable to know the exact location of this real root. This way, an estimate of the degree of accuracy obtained neglecting its effects can be made.

Consider once again the characteristic equation

$$\frac{K_a K_z s^2}{(s+\alpha)(s^2+ps+K_1K_z)} = -1$$

$$K_a = \frac{-(s+\alpha)(s^2+ps+K_1K_z)}{K_z s^2}$$

Let $s=(-a+jb)\omega_{n_4}$ where ω_{n_4} is the final (or selected) natural frequency for the complex roots. Substitute, expand and consider the real part only:

$$K_a = \frac{1}{K_2}\left[a\omega_{nf} - p - \alpha + \frac{(K_1K_2+p\alpha)a}{(a^2+b^2)\omega_{nf}} - \frac{K_1K_2\alpha(a^2-b^2)}{(a^2+b^2)\omega_{nf}^2}\right]$$

but $a^2+b^2 = 1$

$$(1) \quad K_a = \frac{1}{K_2}\left[a\omega_{nf} - p - \alpha + \frac{aK_1K_2}{\omega_{nf}} + \frac{ap\alpha}{\omega_{nf}} - \frac{\alpha K_1K_2(a^2-b^2)}{\omega_{nf}}\right]$$

In which K_1, K_2, P, α are known from the system components and a, b, ω_{nf} are specified by selecting the desired root location on the root locus. If the chosen root location is on the inner circle, this equation must be used, but for a root location on the outer circular arc $K_1K_2 = \omega_{ni}^2 = \omega_{nf}^2$

so that

$$K_a = \frac{1}{K_2}\left[a\omega_{nf} - p - \alpha + a\omega_{nf} + \frac{ap\alpha}{\omega_{nf}} - \alpha(a^2-b^2)\right]$$

and it is apparent that α, $\alpha(a^2-b^2)$ and $\frac{ap\alpha}{\omega_{nf}}$ are all small and may be neglected. Then

$$\ast_1 \quad K_a \cong \frac{1}{K_z}\left(2a\omega_{n_f} - p\right)$$

An alternate derivation of this latter equation is to note from the characteristic equation that

$$K_a K_z + p + \alpha = \sum \text{ROOTS} = 2a\omega_{n_f} + r_3$$

(2) from which $$K_a = \frac{2a\omega_{n_f} - p + r_3 - \alpha}{K_z}$$

and for $r_3 \cong \alpha$ with both α and r_3 small and ω_{n_f} large this readily reduces to the above expression. $\ast_1$

It is interesting to note that combining (1) and (2) gives an exact expression for the location of the real root:

$$r_3 = -a\omega_{n_f} + \frac{aK_1K_z}{\omega_{n_f}} + \frac{ap\alpha}{\omega_{n_f}} - \alpha\frac{K_1K_z(a^2-b^2)}{\omega_{n_f}^{2}}$$

And if the complex roots lie on outer semi circles,

$$r_3 \cong \alpha\left[\frac{ap}{\omega_{n_f}} - a^2 + b\right]$$

Thus, from this nomogram the required values of K_a, α can be obtained for any set of complex roots. Also enough information will be available to find the exact position of the third real root.

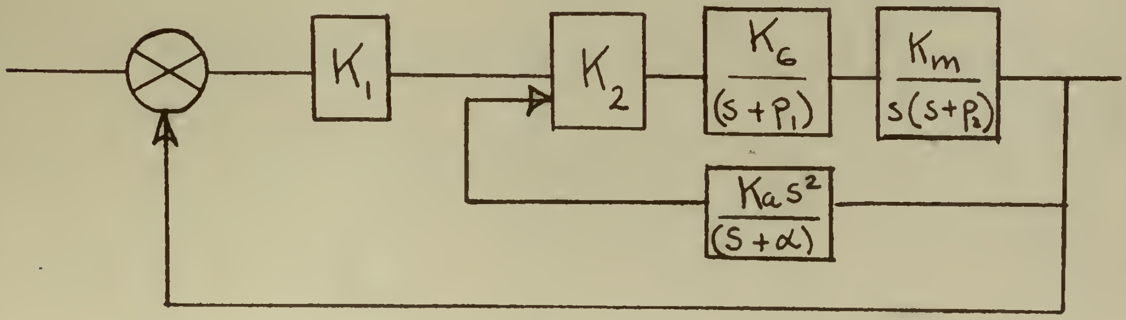
CHAPTER FOUR

ACCELERATION FEEDBACK APPLIED TO THIRD AND HIGHER ORDER SYSTEM

Most control systems are at least third order, therefore a discussion of acceleration feedback compensation would be incomplete, if higher order systems were ignored. With these systems it is not likely that a general conclusion regarding the effect of acceleration feedback will be reached, as it was the second order system. One reason for this lack of generality is that the feedback loop need not enclose all of the poles of the forward loop. The effect of acceleration feedback will then be different for each feedback loop selected. However, the techniques already presented will make a quick analysis of these higher order systems worthwhile. In this way, it can be seen rather rapidly if this type of compensation will be useful in obtaining the desired results when placed in several different feedback paths.

Before starting the discussion pertaining to these higher order systems, it should be pointed out that if the real root of a third order system is large enough to neglect, approximation by second order system calculations may be adequate.

Consider the following third order system.



which has approximate acceleration feedback compensation in a feedback loop that encloses all of the poles of the forward loop.

The characteristic equation of the initial third order system, before compensation can be expressed as

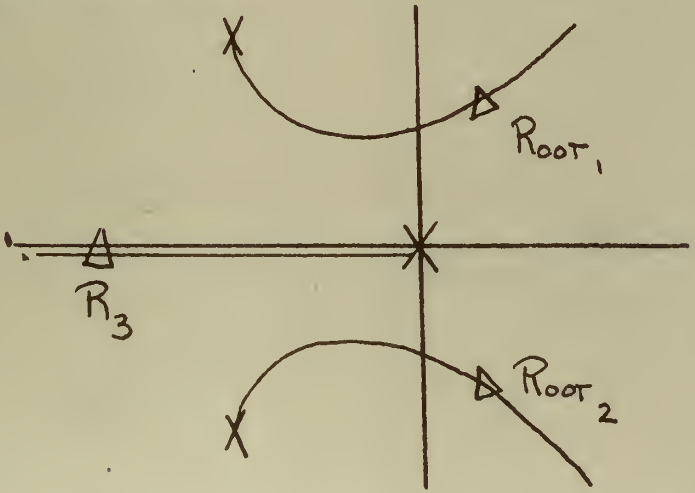
$$1 + KGH = 0$$

$$1 + \frac{K_1 K_2 K_G K_m}{s(s+p_1)(s+p_2)} = 0$$

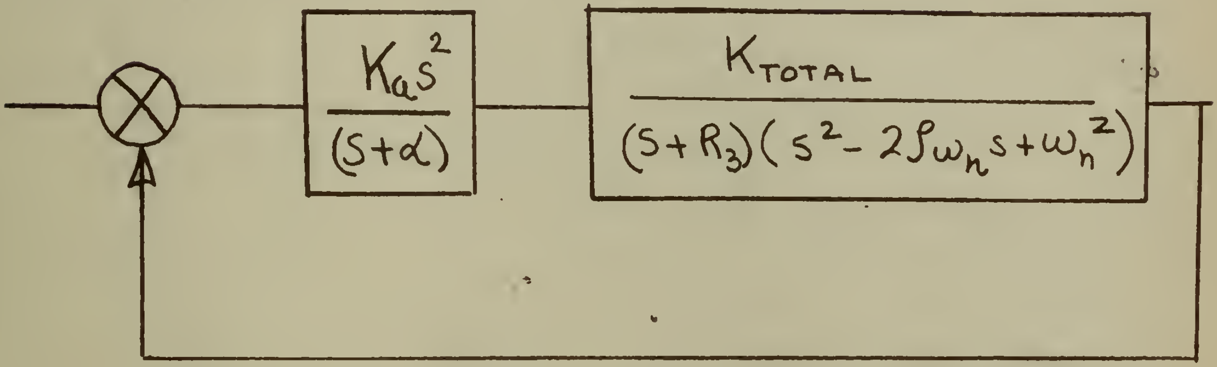
or more commonly

$$1 + \frac{K_1 K_2 K_G K_m}{s(s^2 + 2\rho\omega_n s + \omega_n^2)} = 0$$

where because of high gain the initial roots lie in the right half plane, giving an unstable system to start with. That is, the root locations before compensation are



Employing this type of compensation, and the techniques previously expressed in reducing the system to a compensator in cascade with the overall system, the following is obtained.

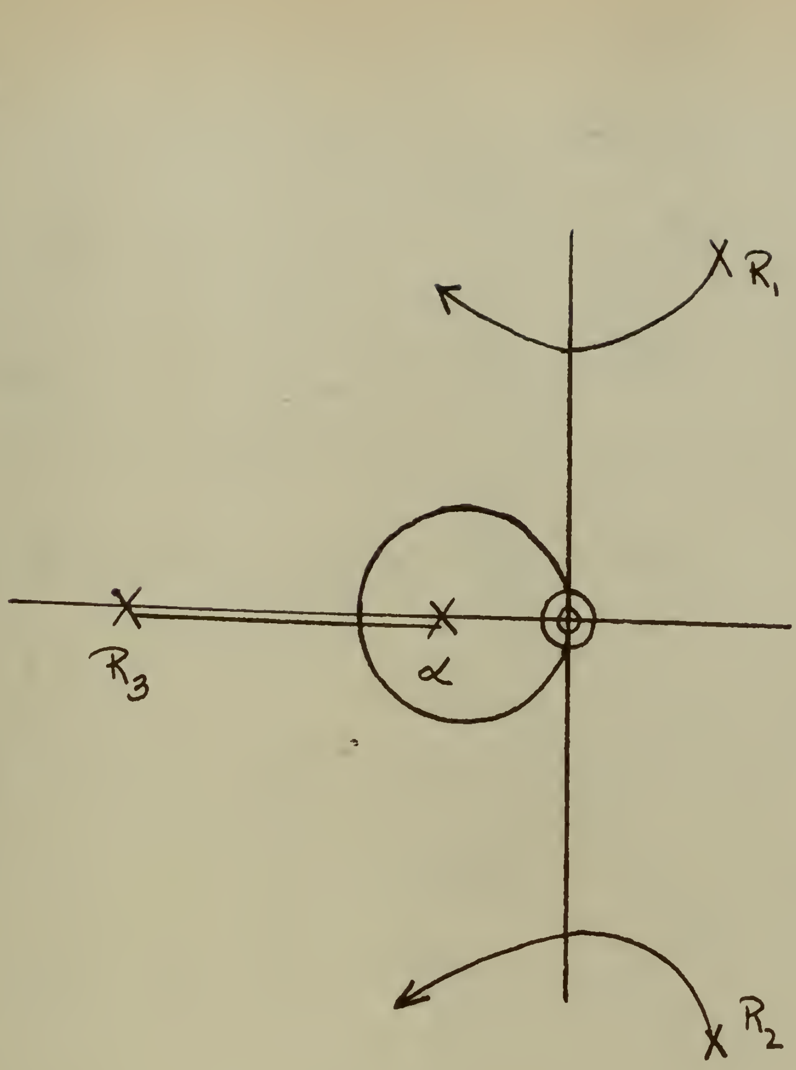

$\frac{K_a s^2}{(s+\alpha)}$
$\frac{K_{TOTAL}}{(s+R_3)(s^2-2\rho\omega_n s+\omega_n^2)}$

$$K_{TOTAL} = K_1 K_2 K_G K_m$$

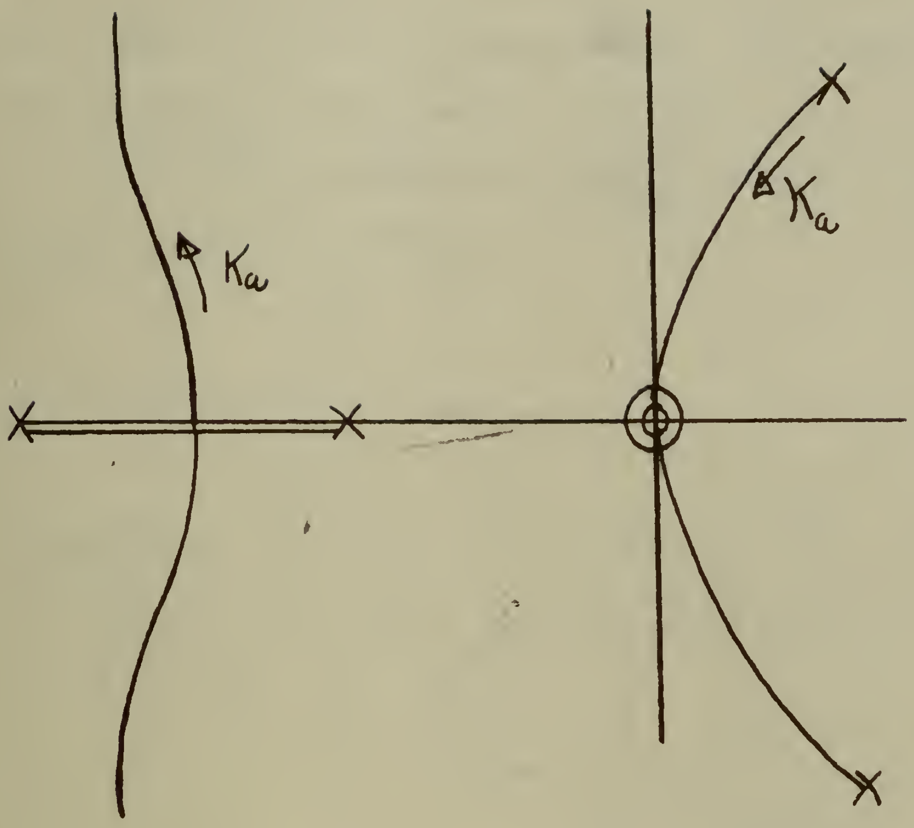
where $(s+R_1)(s+R_2) = s^2 - 2\rho\omega_n s + \omega_n^2 = 0$

so that $1 + KGH = 1 + \dfrac{K_a K_{TOTAL} s^2}{(s+\alpha)(s+R_3)(s^2 - 2\rho\omega_n s + \omega_n^2)} = 0$

therefore



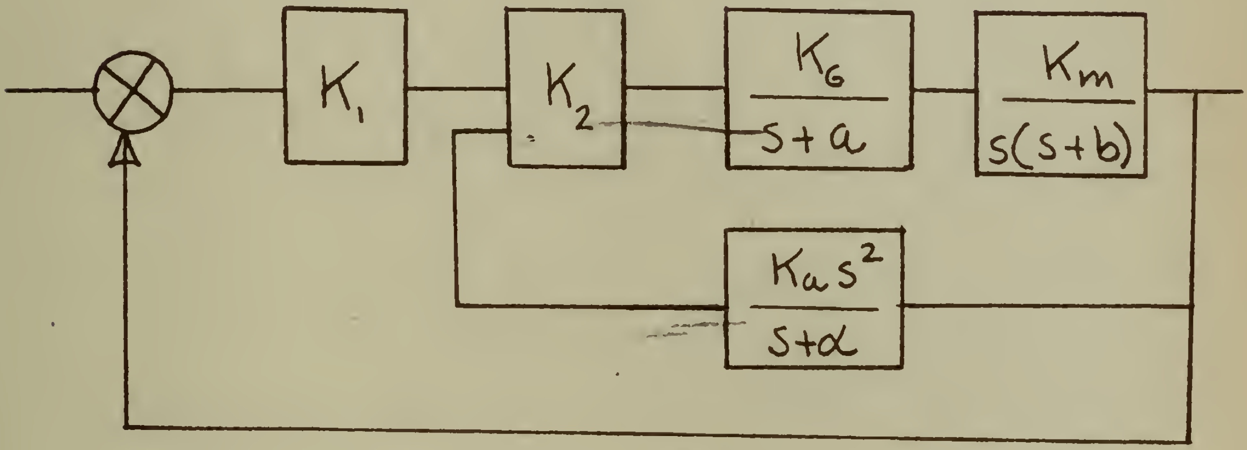
Approximate acceleration feedback may work if the asymptote is in the left half plane, for in this case a pair of complex roots with relatively low ω_n can probably be made to dominate. Note also in this case, that in order to insure stability of the system after compensation, the compensator pole α must be placed near the origin. If it wasn't, the following root locus would result and it is apparent that the system would still be unstable.



In the first case, it can be seen that this type of compensation may make the system not only stable, but place the roots at a desired location, if the acceleration feedback gain is high enough. This case looks rather interesting, for it will be noted that as the gain is increased, one set of roots (those furtherest out from the origin) are moved from the right half plane making the system stable. At the same time, the nearest set of complex roots gets closer and closer to the origin, traveling along what might be perhaps a pure circle. It may turn out that the gain can be selected at such a value that the nearest set of complex roots can be set at desired locations, and the effect

of those farther out can be neglected. It should be pointed out also that for any third order system having the compensator in this feedback loop, a general rule may be obtained for the positioning of the compensator pole and the selection of the approximate gain.

To illustrate this point the following examples are presented. Consider the following system:



Let $(s+a)(s+b) = s^2+4s+8$

and $K_1K_2K_GK_m = 124$

so that the characteristic equation of the initial system is

$$1+KGH = 1+\frac{124}{s(s^2+4s+8)} = 0$$

Then from the root locus plot it can be seen that the initial roots of the system are located at $s = -6, +1 \pm j\,4.5$. The complex roots' presence in the right half of the s plane indicates instability.

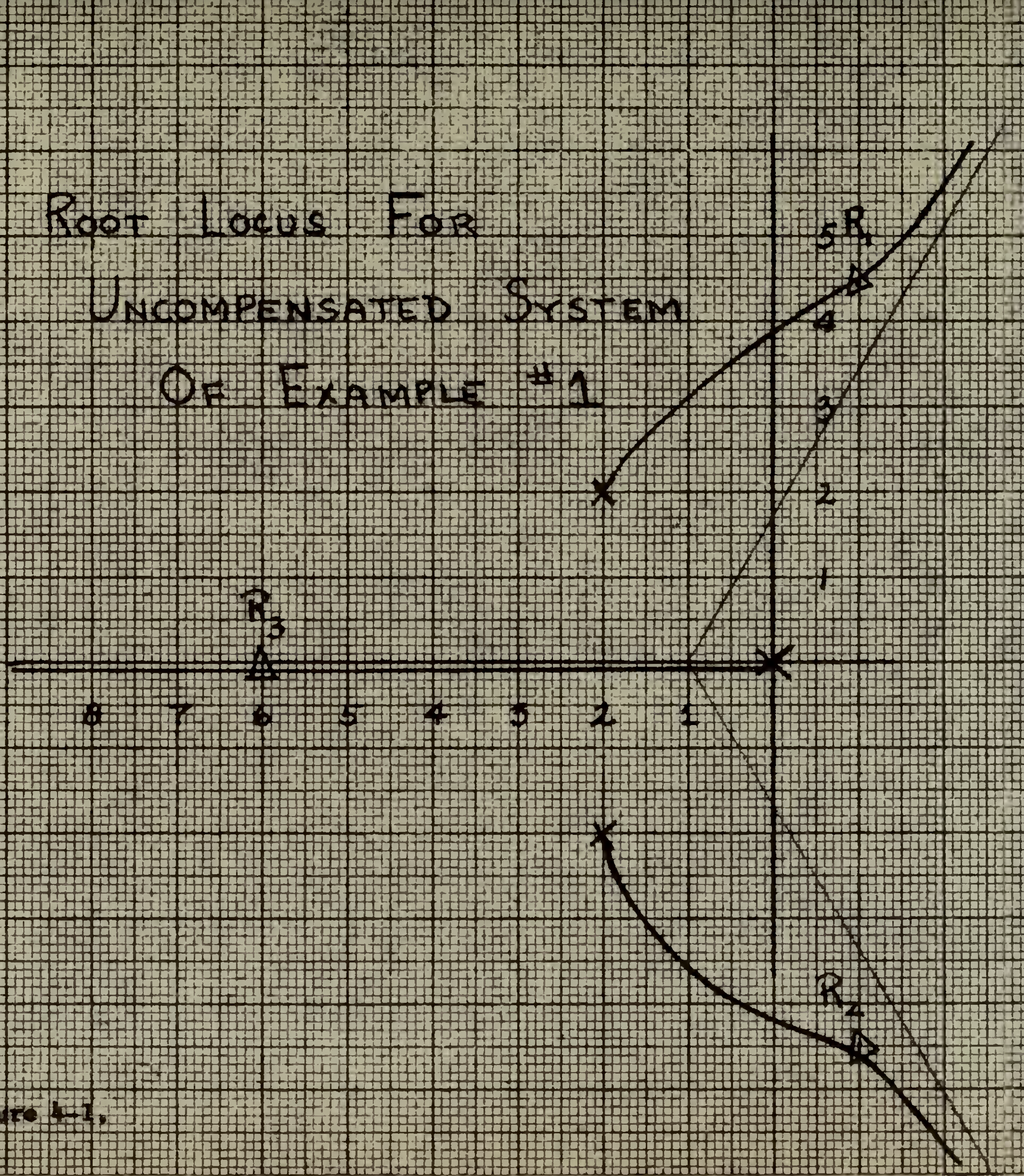

ROOT LOCUS FOR
UNCOMPENSATED SYSTEM
OF EXAMPLE #1
5
R_1
4
3
2
1
R_3
8
7
6
5
4
3
2
1
R_2

Figure 4-1.

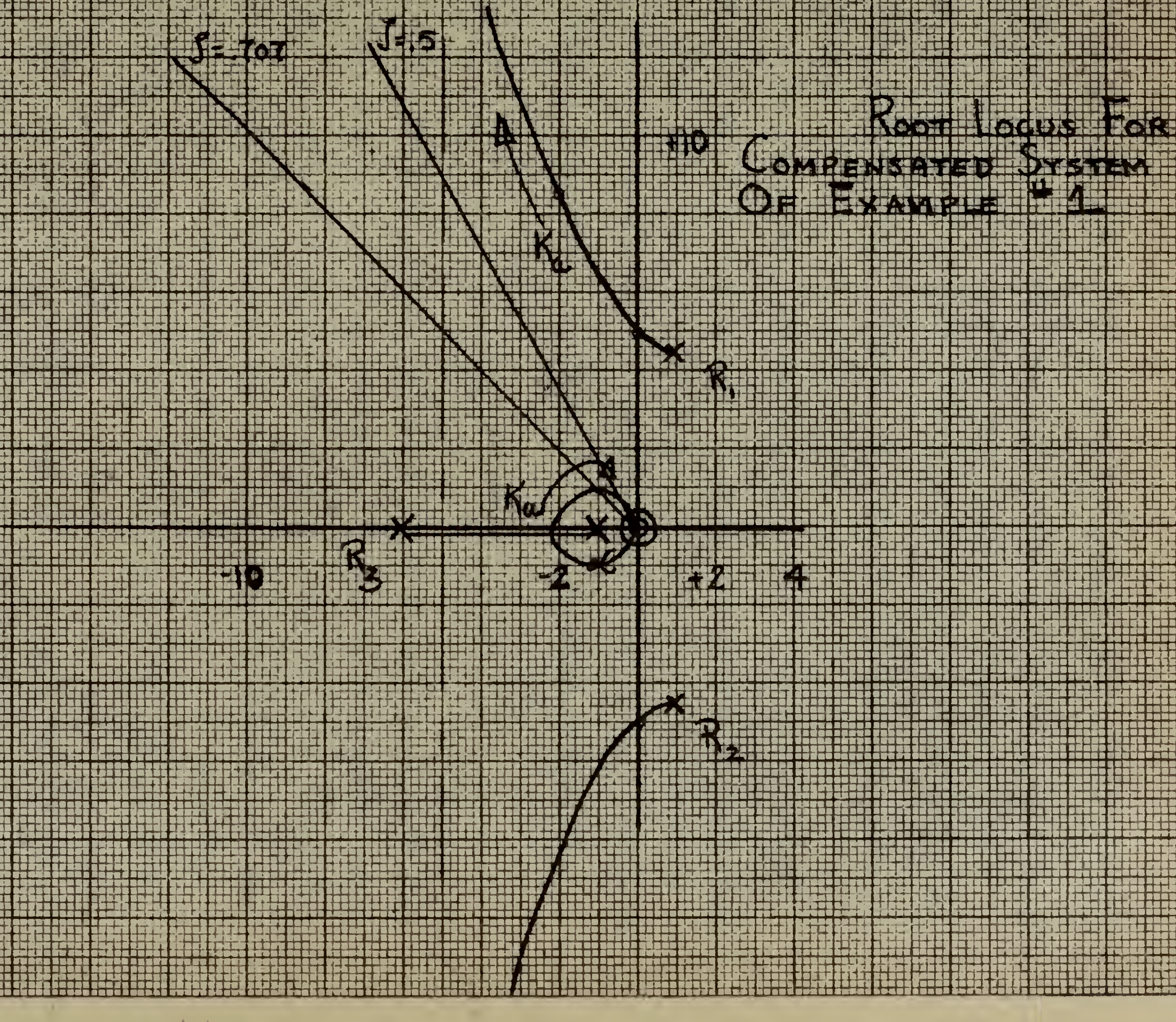

Root Locus For
Compensated System
Of Example # 1
ζ=.707
ζ=.5
+10
Ka
R1
Kω
R3
-10
-2
+2
4
R2

Figure 4-2.

After compensation letting $\alpha = 1.0$, the root locus can be obtained. As it can be seen from the root locus, as K_a is increased the system will be made stable, and if it is desired to place the inner set of complex roots between $\zeta = .72$ and $\zeta = .15$ the other roots will be comparatively further from the origin. However, the effect of these outer roots may be troublesome.

As another example, consider the case where the initial third real root lies very close to the origin. That is, for illustrative purposes let the initial roots (the roots of the uncompensated system) be located at $s = -.5$, $s = -(+5) \pm j5$. Picking the feedback loop so as to enclose all of the feed forward poles once again, the characteristic equation is

$$1 + \frac{K_m K_a s^2}{(s+\alpha)(s^3 + 2\zeta\omega_n s^2 + \omega_n^2 s + K_{TOTAL})}$$

The root locus of this compensated system can be seen to be on the following figure. It should be remembered that since the asymptote angle is 60°, in order for this system to be stable the value of α should be selected so as to insure that the root locus will come into the left half plane. The next thing to consider is the value of gain K_a that will provide first of all stability and then dominance of the nearer complex roots.

As can be seen from the root locus, as K_a is increased the system will be made stable and the desired root location may be selected. The effect of the outer roots may be troublesome here also.

If the feedback loop did not enclose all of the poles of the original system.

ROOT LOCUS PLOT FOR COMPENSATED SYSTEM

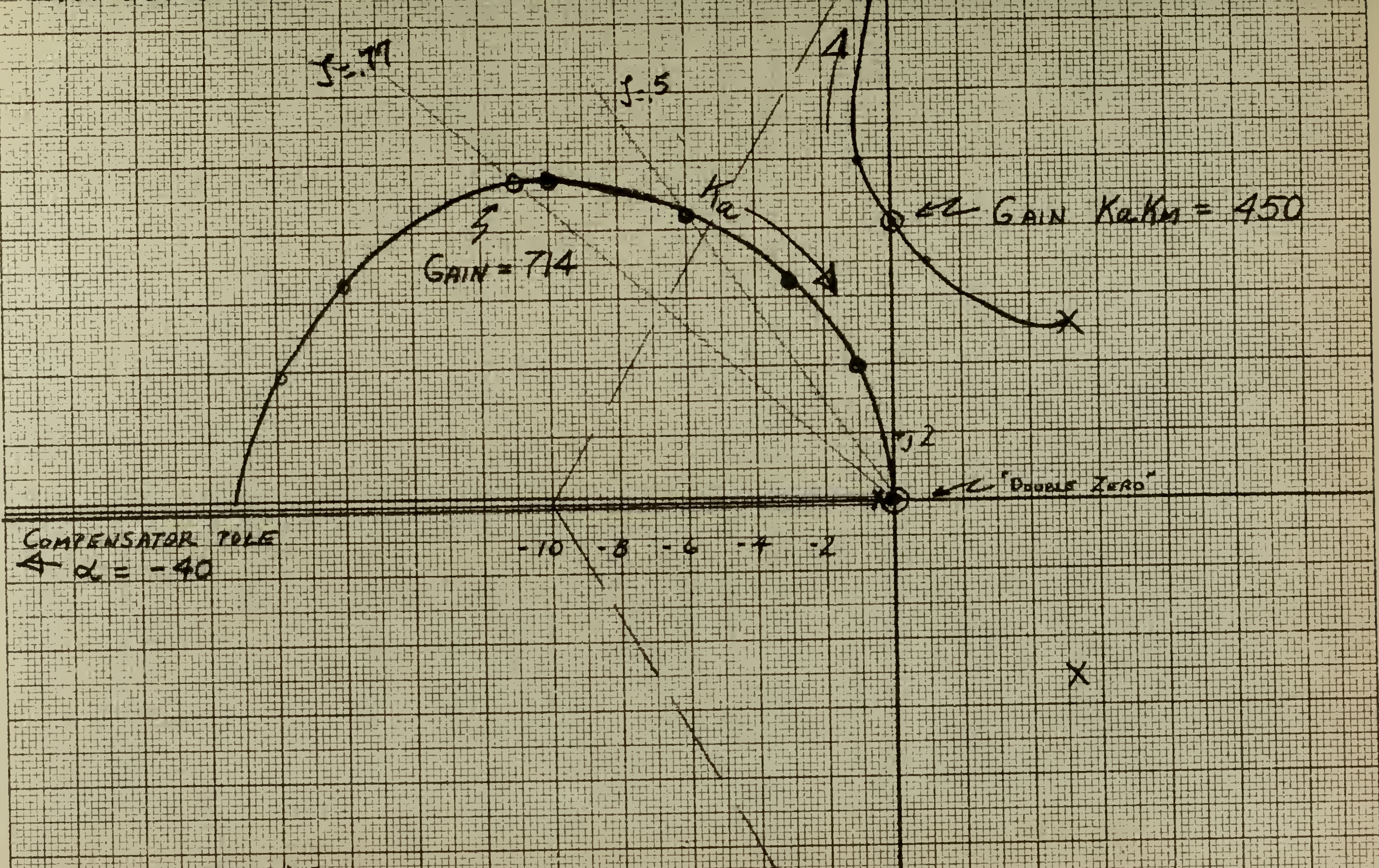
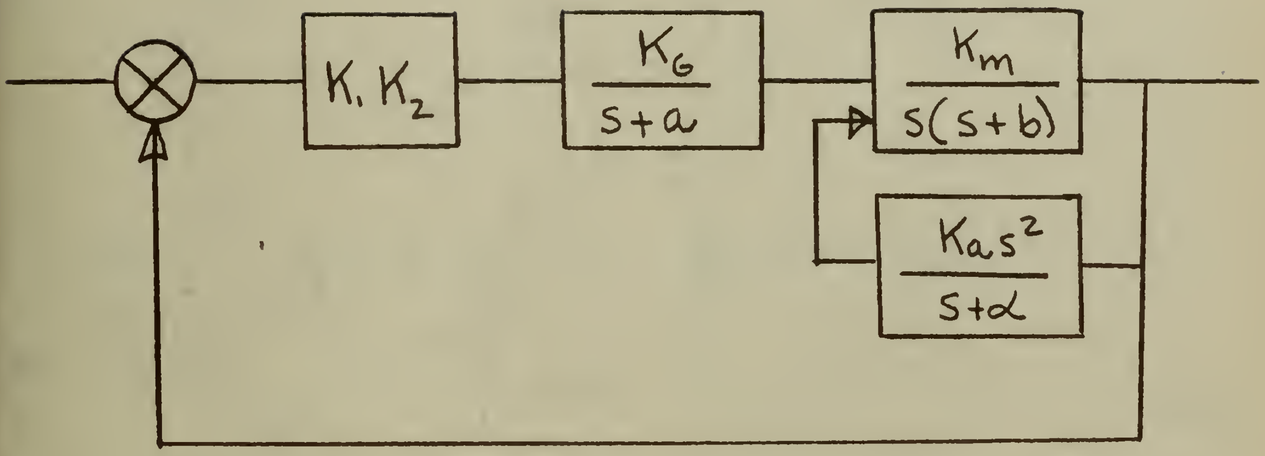
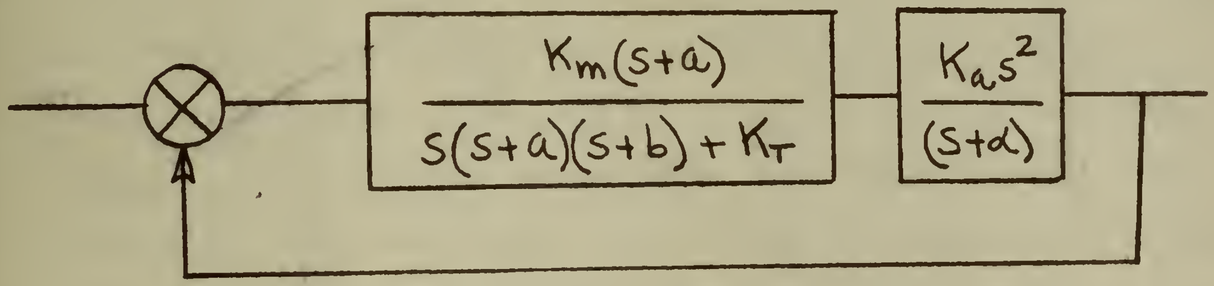


Figure 4-3.

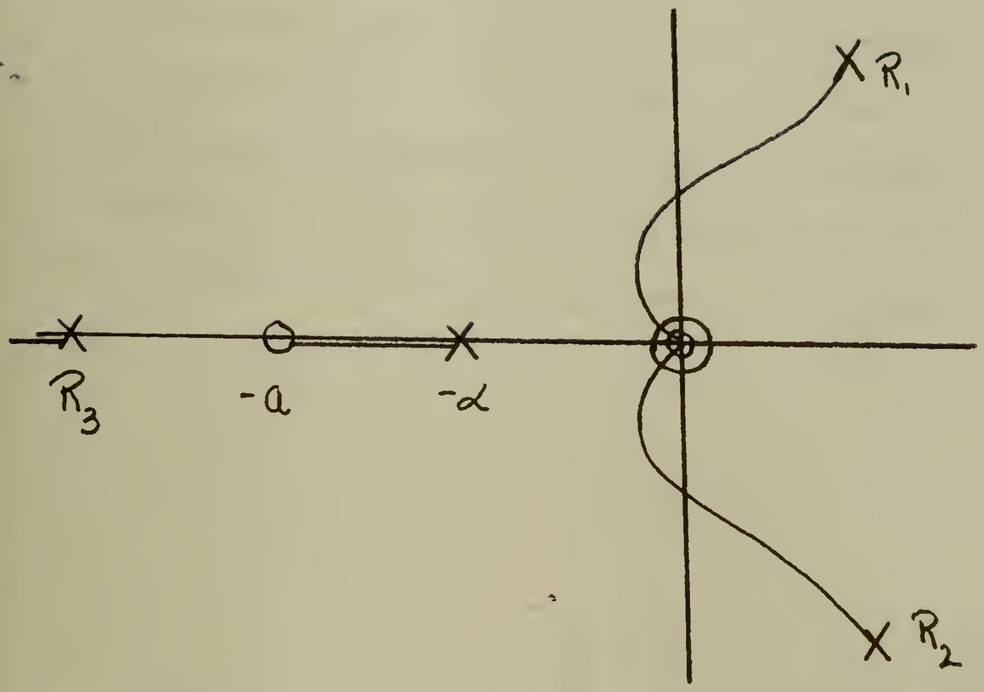


his may be possible for some electronic system where it is permissible to utilize he shown feedback path. If the initial roots existed in the same locations as efore, the new resultant characteristic equation can be determined


$\frac{K_m(s+a)}{s(s+a)(s+b)+K_T}$
$\frac{K_a s^2}{(s+\alpha)}$

$$1 + KGH = 1 + \frac{K_m K_a s^2 (s+a)}{(s+\alpha)\underbrace{\left[s(s+a)(s+b) + K_T \right]}_{\text{Roots of initial system}}}$$

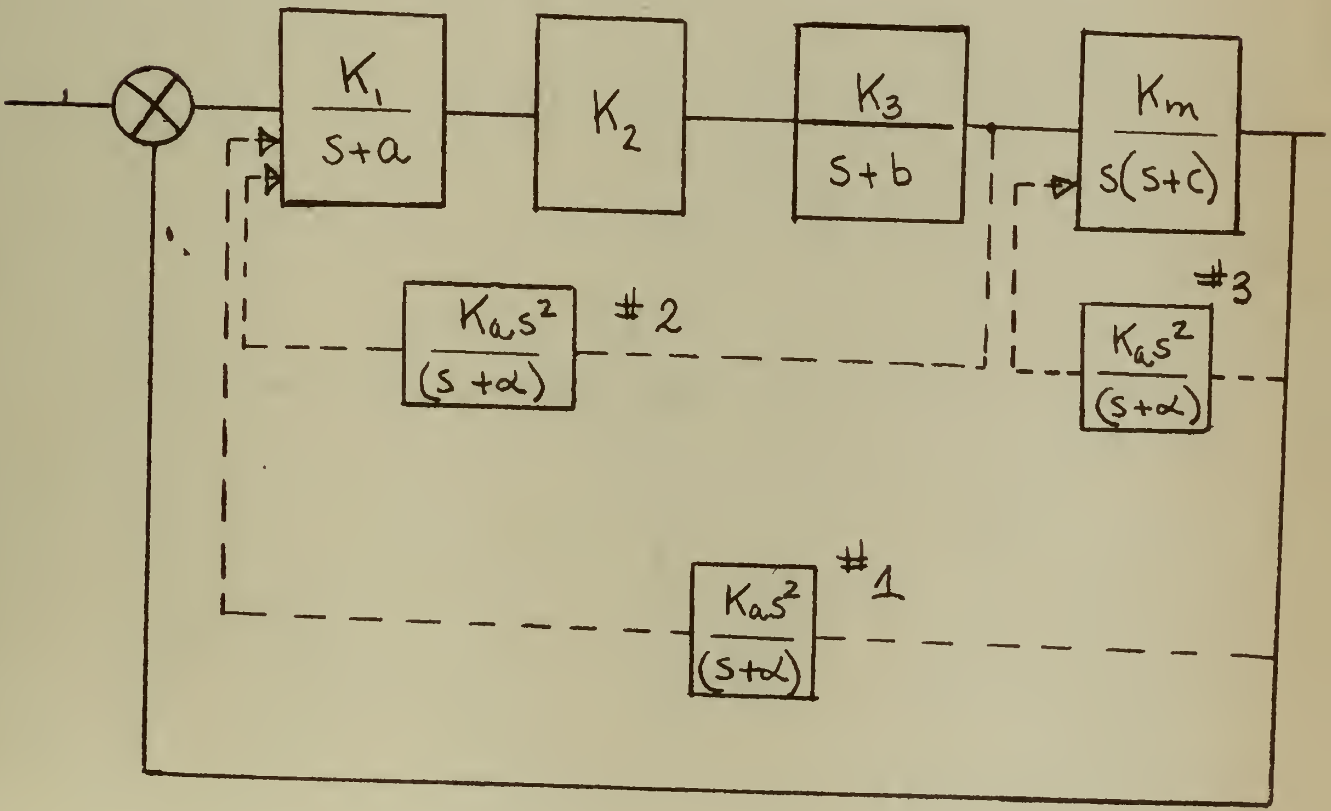
so that



In the above case, one might surmise that there will be many cases where the root locus will never come into LHP leaving the system always unstable. On the other hand if α could be placed close to the origin, then the root locus can reasonably be considered stable if the gain K_a can be made high enough. In the case also, it can be seen that the complex roots can be made dominant if the gain is high enough. However, one problem that might arise in this case is that the complex roots may have a damping ratio ρ that is too low causing a highly oscillatory system. Thus even though the system may be stable, its performance would be undesirable.

HIGHER ORDER SYSTEMS

The same technique may be applied to linear system of any order. The choice of a compensation path depends on the specific application, but need not enclose all of the forward path poles. With this in mind consider the following fourth order system. Immediately the choices for feedback paths more numerous then for any system previously discussed. These choices will, of course, increase as even higher order systems are used.

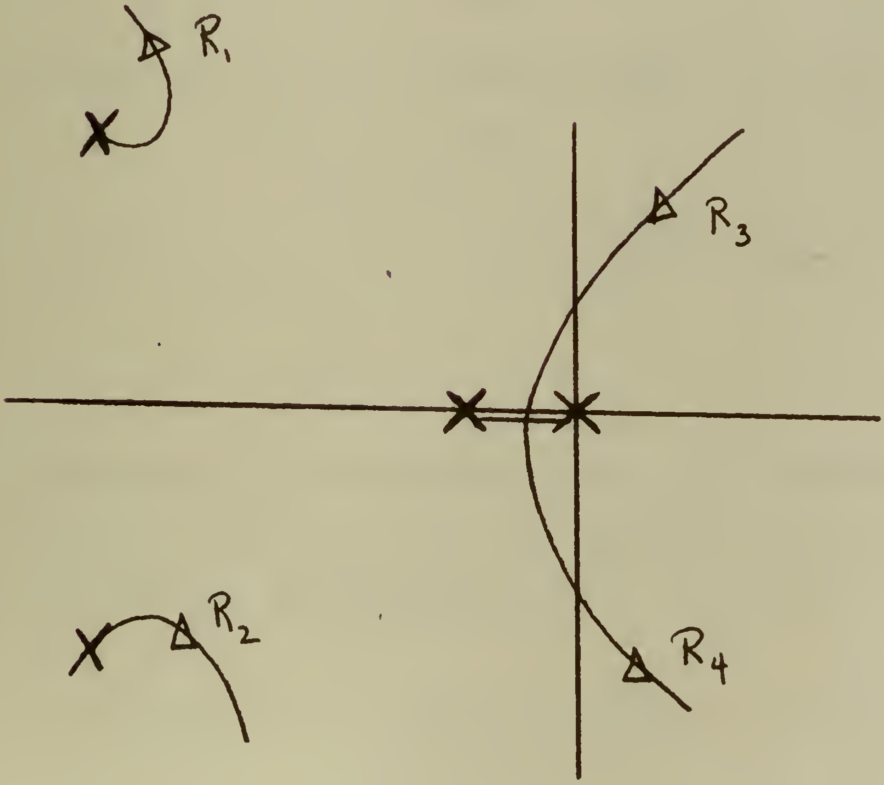

$\frac{K_1}{s+a}$
K_2
$\frac{K_3}{s+b}$
$\frac{K_m}{s(s+c)}$
$\frac{K_a s^2}{(s+\alpha)}$
#2
#3
$\frac{K_a s^2}{(s+\alpha)}$
#1
$\frac{K_a s^2}{(s+\alpha)}$

The above system has an initial characteristic equation of the form

$$1 + KGH = 1 + \frac{K_{TOTAL}}{s(s+a)(s^2 + 2\zeta\omega_n s + \omega_n^2)} = 0$$

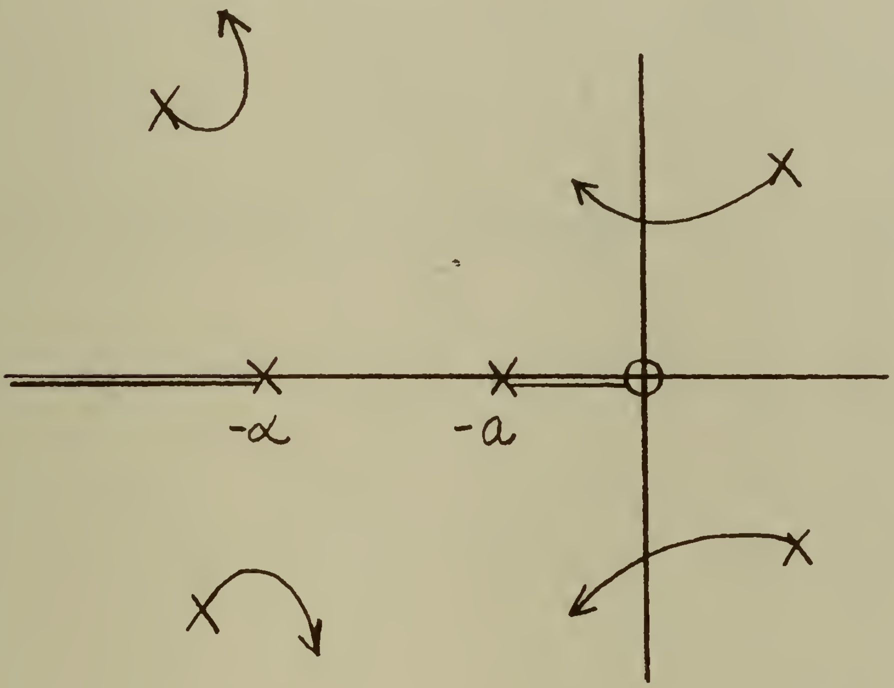
Where $(s+b)(s+c) = s^2 + 2\zeta\omega_n s + \omega_n^2$

and $K_{TOTAL} = K_1 K_2 K_3 K_m$



As is illustrated in the sketch for the root locus of this system without compensation, the system is definitely unstable. Employing compensation so that all the feed-forward poles are enclosed will yield

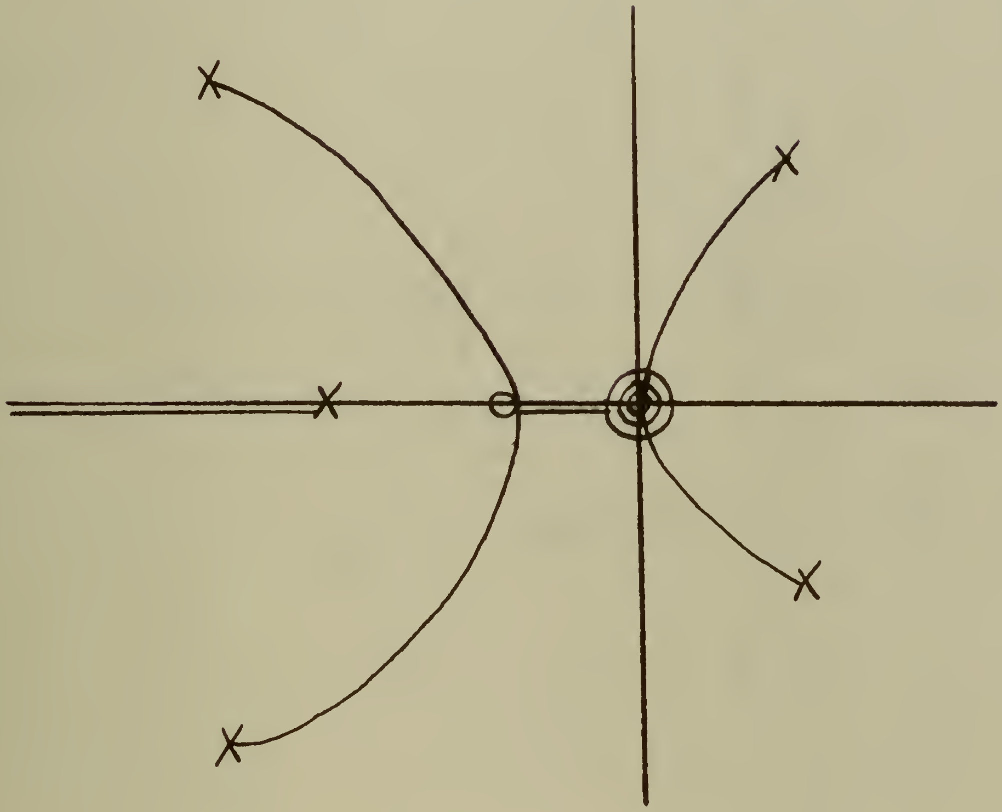
$$1+KGH = 1 + \frac{K_a K_T s^2}{s(s+a)(s+\alpha)(s^2+2\zeta\omega_n s+\omega_n^2)} = 0$$



The system can be made stable as long as the compensator pole and acceleration feedback gain K_a are selected large enough. If possibility #2 is explored the resultant characteristic equation is

$$1 + KGH = \frac{K_a K_1 K_2 K_3 s^2 s(s+c)}{(s+\alpha)\left[s(s+a)(s+b)(s+c)+K_T\right]} = 0$$

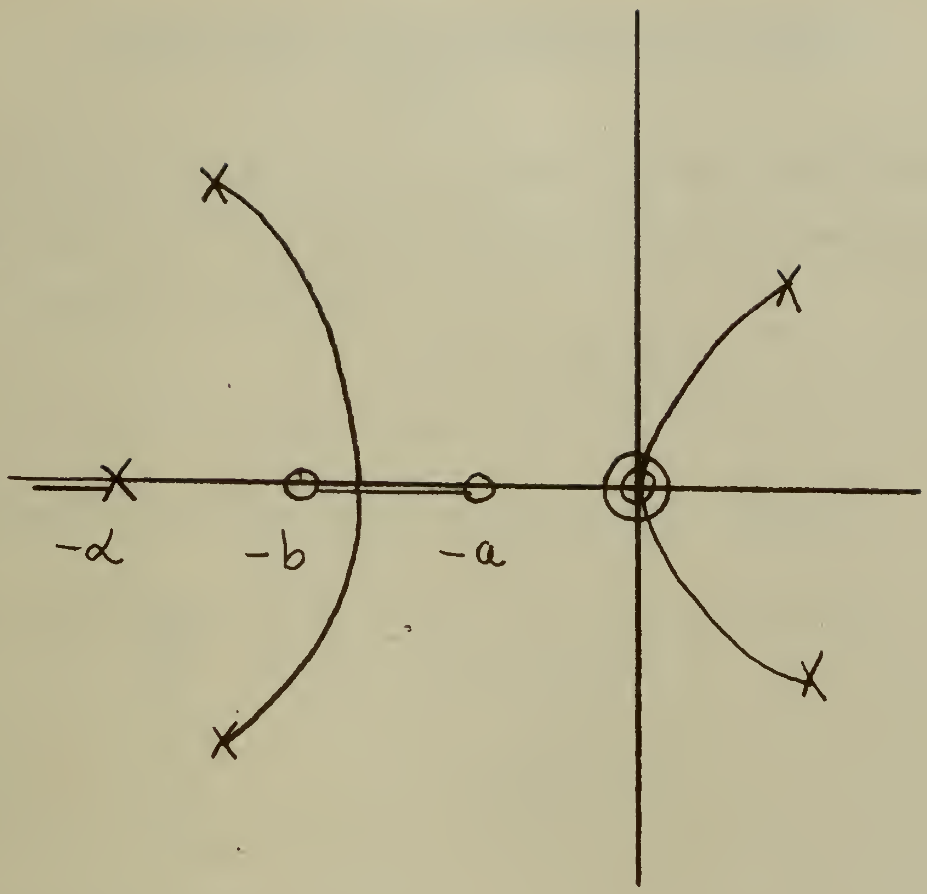
In this case, the system will remain unstable in spite of the presence of the compensator.



Finally if the third possibility is selected, the characteristic equation of the compensated system will be

$$1 + KGH = 1 + \frac{K_a K_m s^2 (s+a)(s+b)}{s(s+c)(s+b)(s+a)(s+\alpha) + K_T(s+\alpha)} = 0$$

so that



the system will remain unstable in this case also.

In the preceding illustrations, it was hoped that perhaps some general rule could be worked out, however this is difficult for the higher order system. It is believed however that the type of compensation developed in this thesis will be an aid in the compensation of many higher order systems. For these systems, the effects of this compensator can be checked rather easily, and will be unique for each system and each feedfack path selected for the compensator. In this chapter the higher order systems mentioned and analyzed were the third and fourth order. The expansion of this analysis to even higher order systems can be made without difficulty, but since the importance of such an analyses would be warranted only for specific system, it is not presented here.

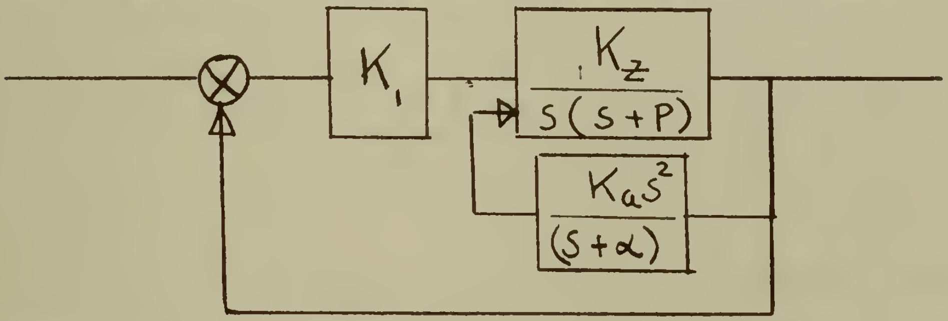
APPENDIX A

DETERMINATION OF BREAKAWAY POINTS AND ESTABLISHMENT OF ROOT LOCUS FOR THE COMPENSATED SECOND ORDER SYSTEM

In inserting a filter in the feedback loop and feeding the output of a first derivative device into it, the characteristic transfer function of the feedback loop becomes

$$KG_{\text{FEEDBACK LOOP}} = \frac{K_a s^2}{(s+\alpha)}$$

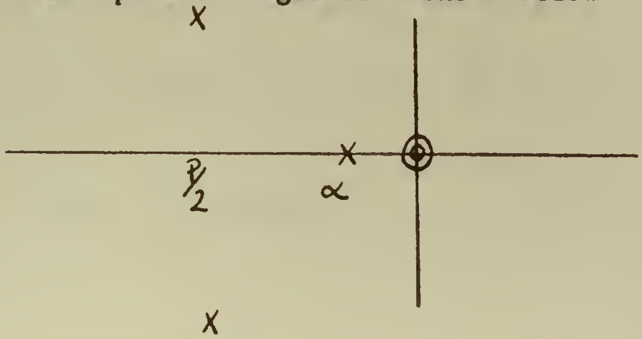
So that if a second order system is considered of the form

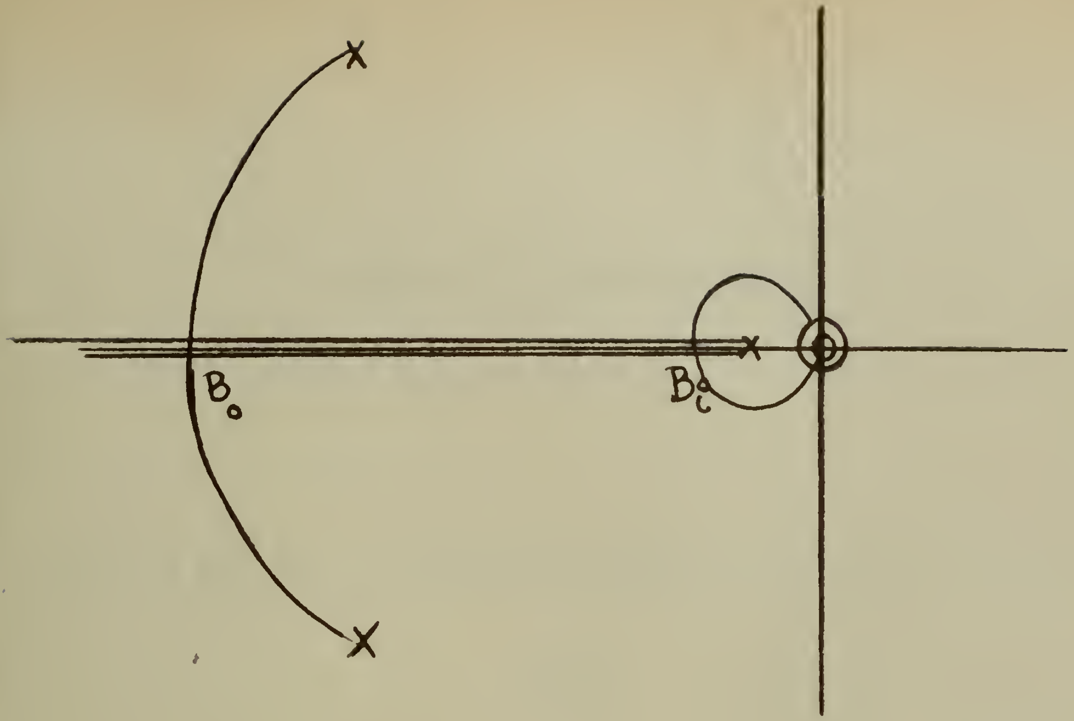


the characteristic equation is:

$$1+KGH = 1 + \frac{K_a K_z s^2}{(s+\alpha)(s^2+ps+K_1K_z)} = 0$$

This gives the zero and pole configuration shown below



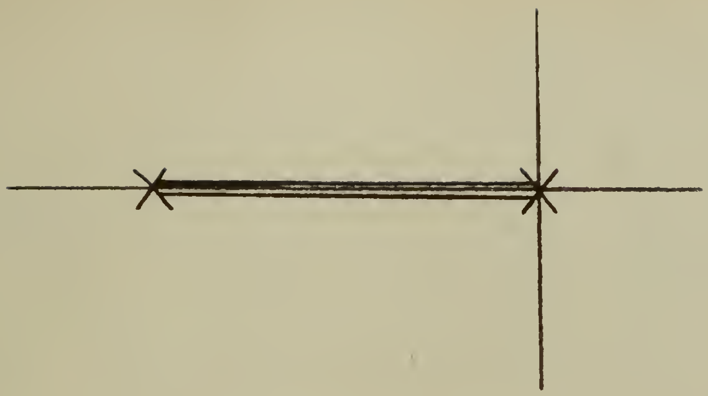


If the breakaway point is desired there are several ways it may be obtained. One method is to consider the fact that the plot of gain versus distance from the origin (in the negative real axis direction only) yields a maximum gain at breakaway points and a minimum gain at the break-in points. That is, since

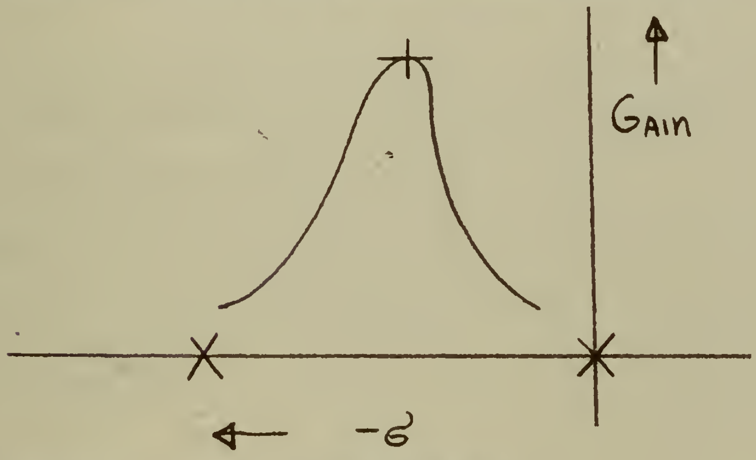
$$\text{GAIN AT ROOTS} = \frac{\text{PRODUCT OF POLE DISTANCES}}{\text{PRODUCT OF ZERO DISTANCES}}$$

where by pole and zero distance it is meant the distance from the considered root to the respective pole or zero.

Consider the simple case where the pole--zero configuration is

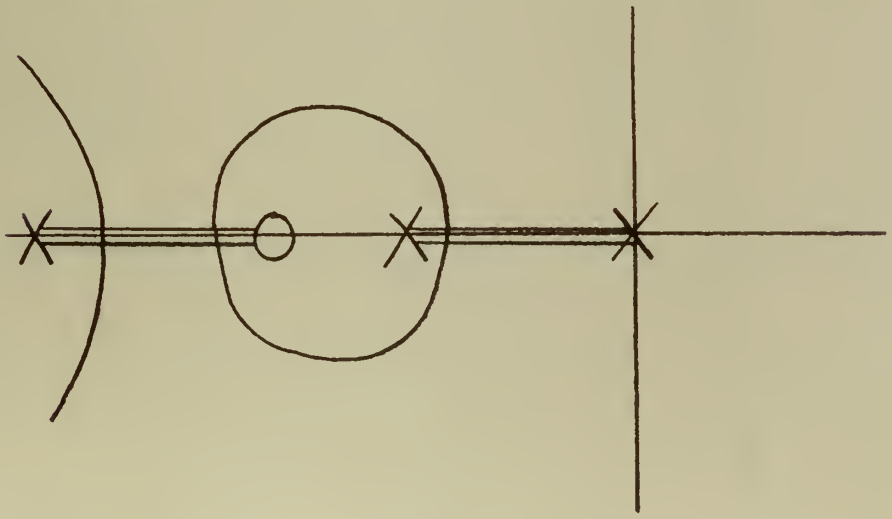


A plot of Gain (of roots) versus distance from origin will yield

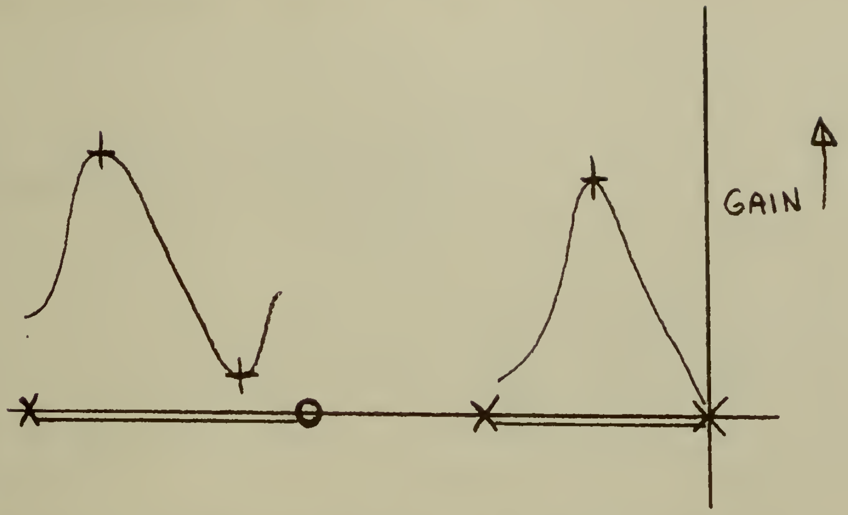


and breakaway will occur at maximum gain.

For the zero-pole configuration shown below

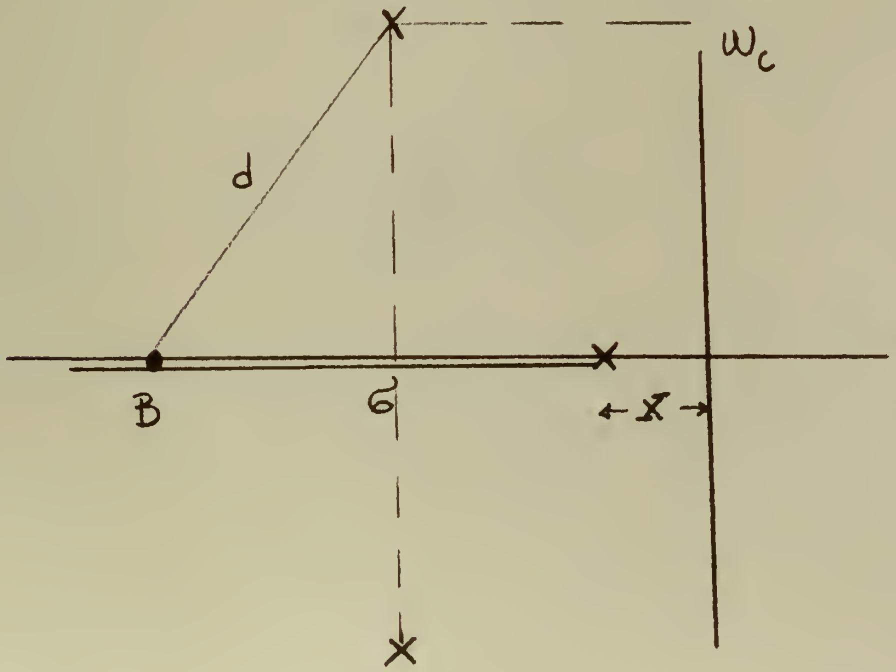


The gain (of roots) plot will yield



which clearly illustrates that maximum gain indicates breakaway points whereas minimum gain indicates breaking points.

Now utilizing this idea consider the zero, - pole configuration of interest once again to determine the limiting case as far as the breakaway point is concerned.



With the distances from the origin designated by the letters indicated above, it follows.

$$\text{GAIN} = K_G = \frac{(B-x)d^2}{B^2}$$

$$= B^{-2}(B-x)\left[\sqrt{(B-\sigma)^2+\omega_c^2}\right]^2$$

$$= B^{-2}(B-x)\left[(B-\sigma)^2+\omega_c^2\right]$$

$$\therefore \frac{dK}{dB} = B^{-2}(B-x)\left[2(B-\sigma)\right] + \left[(B-\sigma)^2+\omega_c^2\right]\left[\frac{B^{-2}-(B-x)2B}{B^4}\right]$$

$$\frac{dK}{dB} = \frac{2(B-x)(B-\sigma)}{B^2} + \frac{\left[(B-\sigma)^2+\omega_c^2\right]\left[-B^2+2Bx\right]}{B^4}$$

$$\frac{dK}{dB} = \frac{2B^2\left[B^2-(\sigma+x)B+\sigma x\right]}{B^4} + \frac{\left[B^2-2\sigma B+\sigma^2+\omega_c^2\right]\left[-B^2+2Bx\right]}{B^4}$$

To find maximum gain set $\frac{dK}{dB} = 0$

$$\therefore \frac{dK}{dB} = B^4 + B^3\left[2\sigma+2(-\sigma-x)+2x\right] + B^2\left[2\sigma x-(\sigma^2+\omega_c^2)-4\sigma x\right]$$
$$+ (\sigma^2+\omega_c^2)\,2xB = 0$$

This will give the following general expression

$$B^4 - B^2\left[\sigma^2+\omega_c^2+2\sigma x\right] + (\sigma^2+\omega_c^2)\,2xB = 0$$

The roots of this equation will yield the desired breakaway points. There are several methods available to find these roots, but the root locus is perhaps the easiest. Therefore the roots depicted on root locus diagram are the locations of the breakaway points. If these roots are complex conjugates, the breakaway points are nonexistent.

Therefore

$$\frac{(\sigma^2+\omega_c^2)\,2x\,B}{B^4 - B^2\left[\sigma^2+\omega_c^2+2\sigma x\right]} = -1$$

$$\frac{2x(\sigma^2+\omega_c^2)\,B}{B^2\left[B^2-(\sigma^2+\omega_c^2+2\sigma x)\right]} = -1$$

Taking a specific example to illustrate the method to be employed, let

$$(\sigma^2+\omega_c^2) = 2 = \omega_n^2$$

This condition sets the initial roots at fixed locations

$$s = -1 \pm j\,1$$

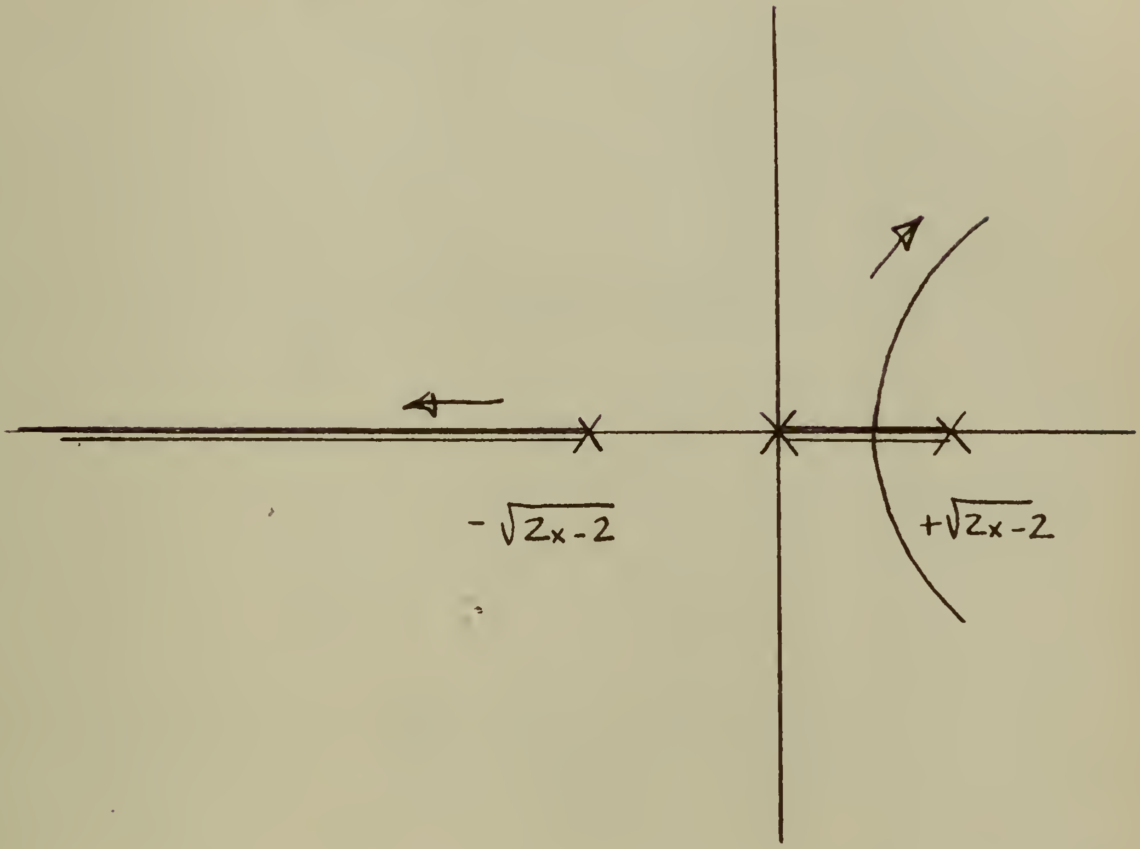
If this is true, the general expression becomes

$$\frac{4x\,B}{B^2\left[B^2-2x-2\right]} = -1$$

And since we know that X will always lie in the LHP (i.e., have a negative value)

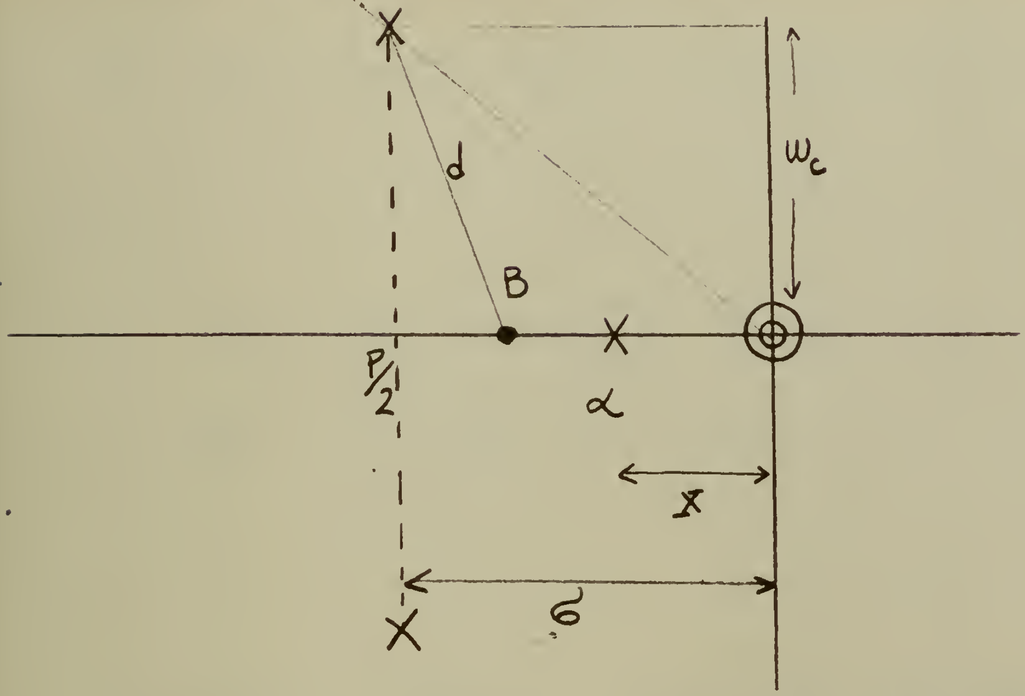
$$\frac{4x\,B}{B^2\left[B^2+(-2x-2)\right]} = -1$$

$$\therefore \quad \frac{4xB}{B^2\left[B \pm \sqrt{2x+2}\right]} = -1$$



Then depending on the value of 4X, the roots of this equation can be found. These roots will enable one to see if there is a breakaway point; and if there is, it will yield its exact location.

If the breakaway point lies to the right of P/2



$$K = \frac{(B-x)\,d^2}{B^2} = \frac{(B-x)\left[(\sigma-B)^2+\omega_c^2\right]}{B^2}$$

$$\frac{dK}{dB} = \frac{(B-x)}{B^2}\left[-2(\sigma-B)\right] + \left[(\sigma-B)^2+\omega_c^2\right]\left[\frac{B^2-(B-x)2B}{B^4}\right]$$

$$0 = -B^2\left[B-x\right]2(\sigma-B) + \left[(\sigma-B)^2+\omega_c^2\right]\left[B^2-(B-x)2B\right]$$

$$= -2B^2\left[-B^2+B(\sigma+x)-\sigma x\right] + \left[\sigma^2+\omega_c^2-B^2-2B\sigma\right]\left(B^2-2B^2+2Bx\right)$$

$$= B^4 - B^2\left(\sigma^2+\omega_c^2+2\sigma x\right) + B\left[2x\left(\sigma^2+\omega_c^2\right)\right]$$

This gives the sums expression as was obtained before.

That is,

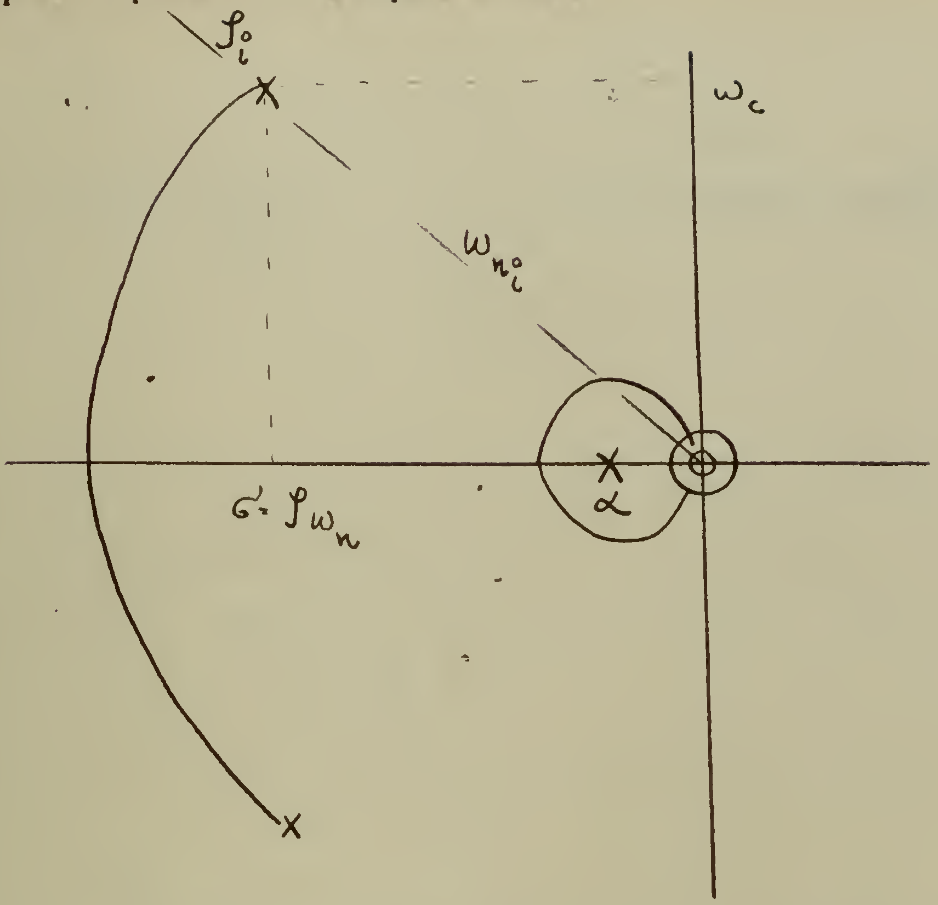
$$B^4 - B^2\left[\sigma^2+\omega_c^2 \; +2\sigma x\right] + B\left[2x\left(\sigma^2+\omega_c^2\right)\right] = 0$$

Therefore

$$\frac{B\left[\,2x\;\left(\sigma^2+\omega_c^2\right)\right]}{B^2\left[\,B^2-\left(\sigma^2+\omega_c^2\right)+2\sigma x\right]} = -1$$

For the case of interest, it is expected that the breakaway points will indicate that the root locus consists of pure semi-circles having the origin at the center of the circle and radius equal to initial ω_n, and circles having the compensator pole as the center and radius equal to distance of compensator pole from origin. To prove this true or indicate the deviation from what is expected, the following analysis was performed.

Examples to prove locii are pure circles:

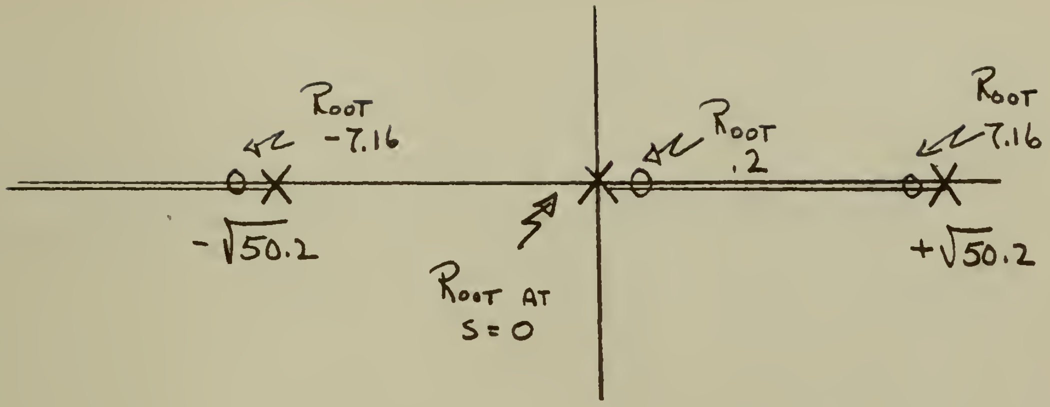


Case #1a) $\alpha = .10$, $\sigma = 1.0$, $\omega_c = 7.0$, $\rho_i^{\bullet} = .15$

$$\therefore \quad B^4 + B^2\left[-2\sigma\bar{x} - (\sigma^2+\omega_c^2)\right] + (\sigma^2+\omega_c^2)\,2x\,B = 0$$

$$B^4 + B^2\left[-50.2\right] + 10B = 0$$

$$\frac{10B}{B^2\left[B^2-50.2\right]} = -1$$



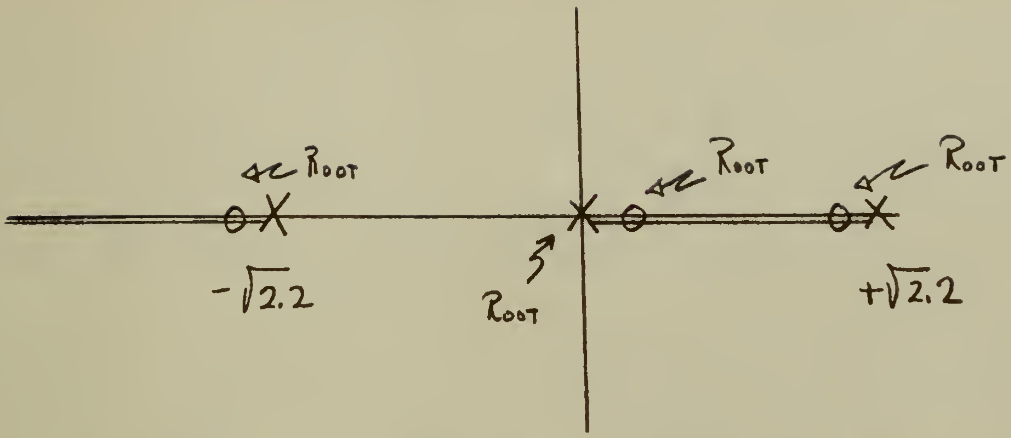
For a gain of 10 the roots of the general expression which hold for both B_o and B_i gives the following roots; s = 7.16, + 7.16, + .2. Since positive numbers were used, $B_o = 7.16$, $B_i = .2$. These results are very close to those expected.

Case 1b) $\alpha = .10$, $\sigma = 1.0$, $\omega_c = 1.0$, $\zeta_i = .707$

$$\therefore \quad B^4 + B^2\left[-2\sigma\alpha - (\sigma^2 + \omega_c)^2\right] + (\sigma^2 + \omega_c^2)\, 2\alpha B = 0$$

$$B^4 - 2.2B^2 + .4B = 0$$

$$\frac{.4B}{B^2\left[B^2 - 2.2\right]} = -1$$



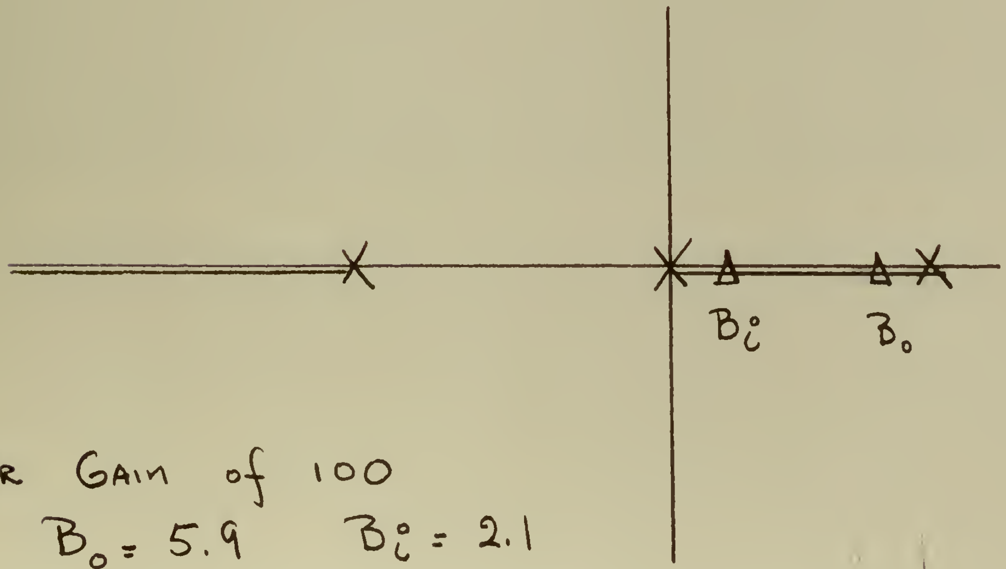
For gain of .4, B_o = 1.41 and B_i = .2. Therefore this checks also.

Case 2a) $\alpha = 1.0$ $\sigma = 1.0$ $\omega_c = 7.0$ $\zeta_i = .15$

$$B^4 + B^2\left[-2\sigma x - (\sigma^2+\omega_c^2)\right] + (\sigma^2+\omega_c^2)\,2xB = 0$$

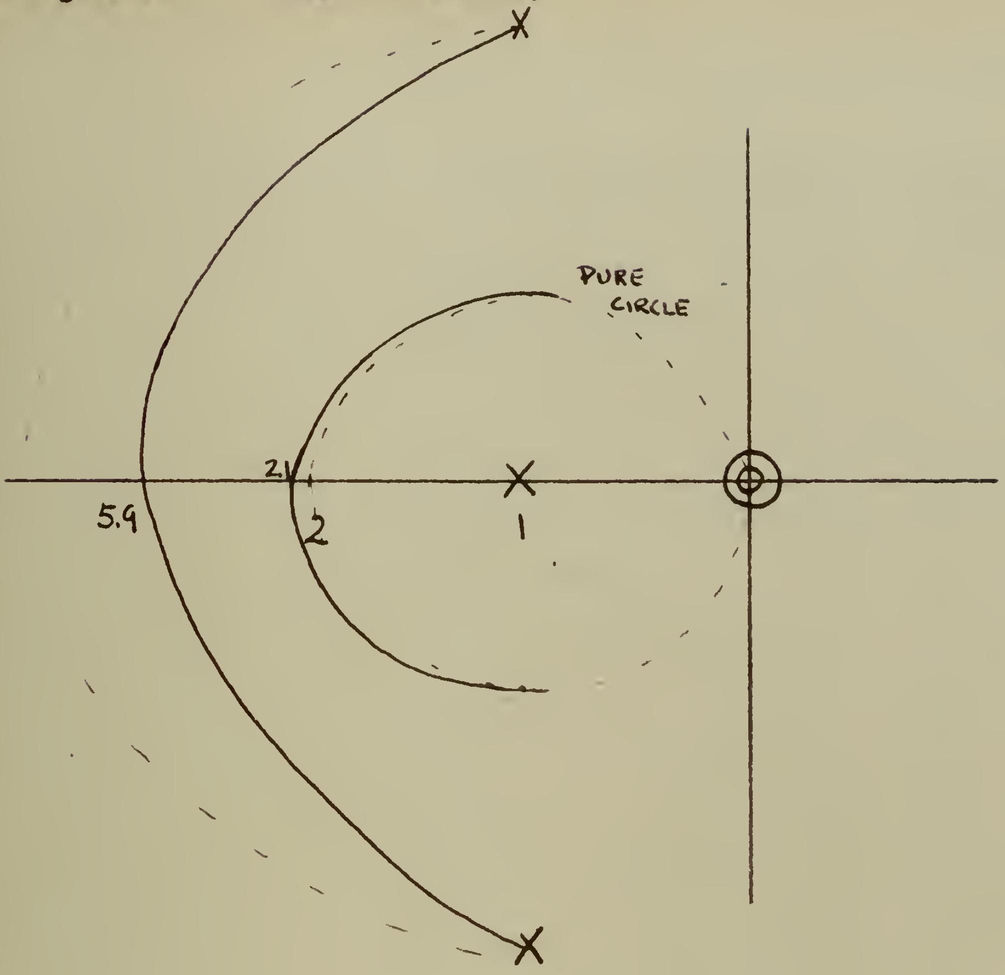
$$B^4 + B^2[-52] + 100B = 0$$

$$\frac{100B}{[B^2-52]B^2} = -1$$



for Gain of 100
$B_o = 5.9$ $B_i = 2.1$

This will give a root locus of the form:



This proves that as α goes farther from the origin the locus must deviate somewhat from a semi circle for the outer locus, however for the inner locus the result is still close to a circle of radius = α with α the center of this circle.

Case 2b) $\alpha = 1.0$, $\sigma = 1.0$, $\omega_c = 1.0$ $\zeta_i = .707$

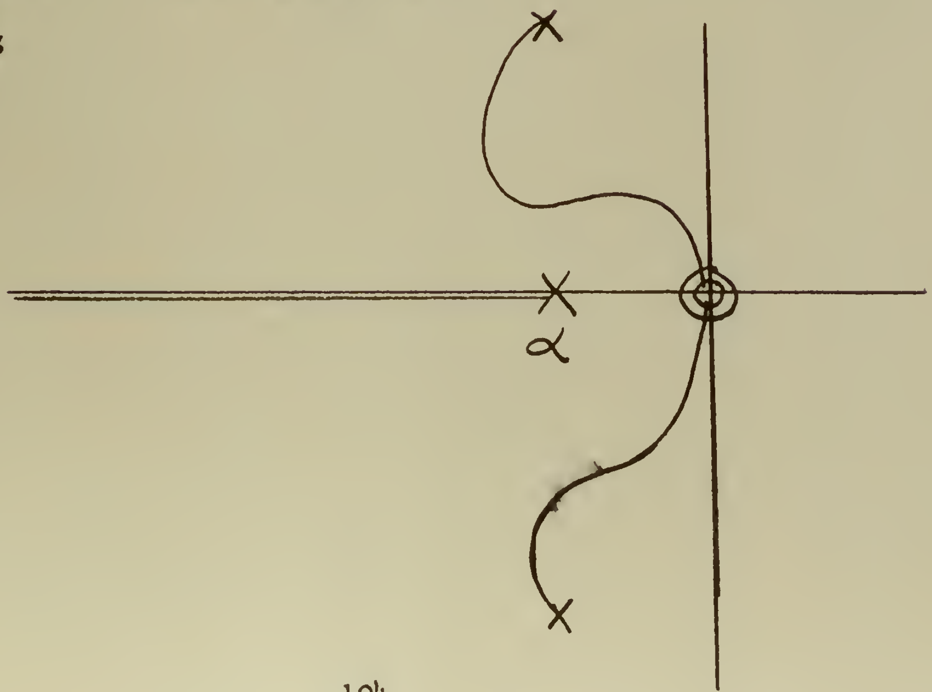
$$B^4 + B^2\left[-2\sigma x - (\sigma^2+\omega_c^2)\right] + (\sigma^2+\omega_c^2)2xB = 0$$

$$B^4 - 4B^2 + 4B = 0$$

$$\frac{4B}{B^2\left[B^2-4\right]} = -1$$

For gain of 4, the roots are complex

giving



which proves that the breakaway point does not exist since the roots are complex.

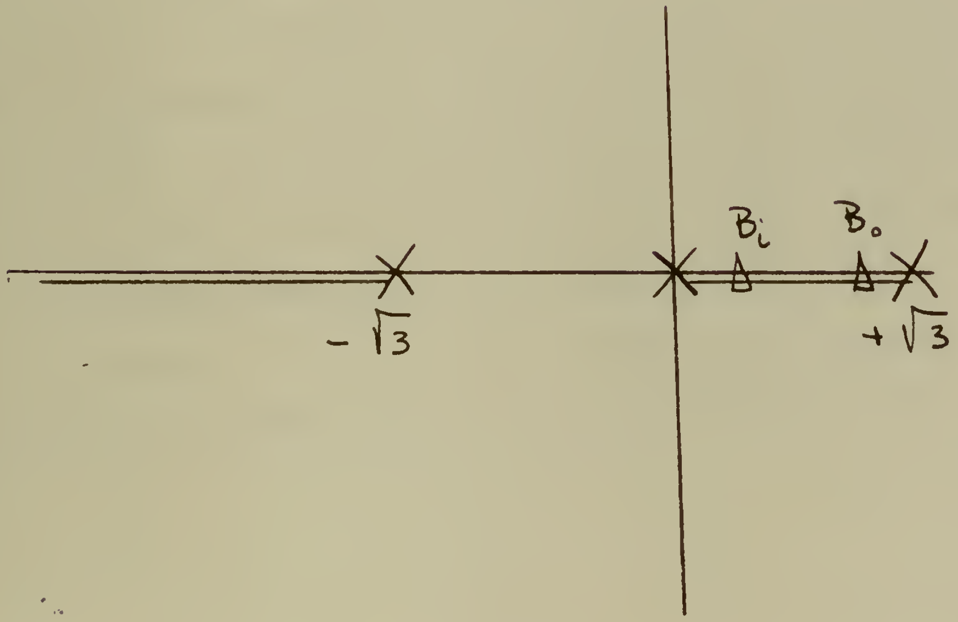
This checks out with what is expected and with the theory given previously.

Finally consider Case 3a) $\alpha = .50$, $\sigma = 1.00$, $\omega_c = 1.00$

$$B^4 + B^2\left[-2\sigma\alpha - (\sigma_2 + \omega_c^2)\right] + (\sigma^2 + \omega_c^2)\,2\alpha B = 0$$

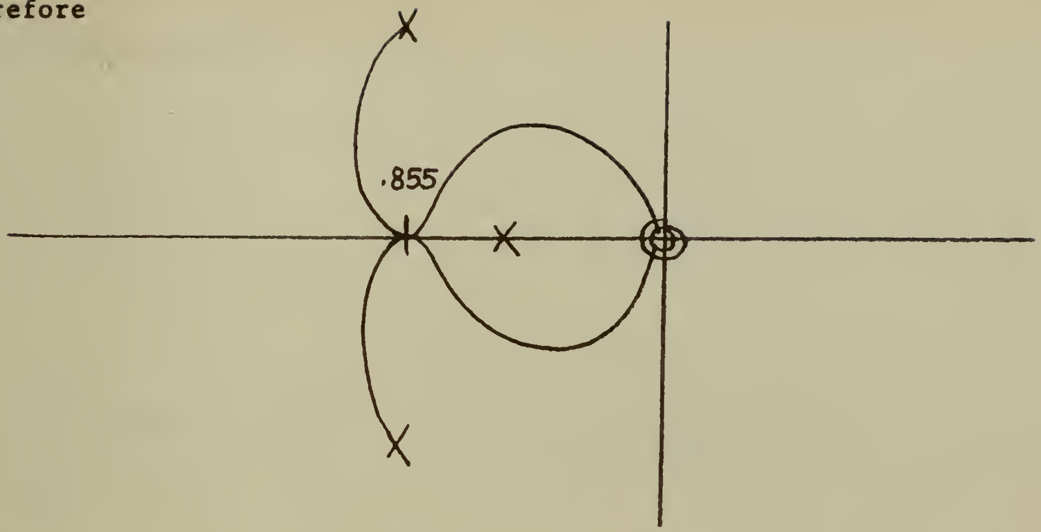
$$B^4 - 3B^2 + 2B = 0$$

$$\frac{2B}{B^2\left[B^2 - 3\right]} = -1$$



For gain of 2, $B_o = B_i = .855$

Therefore

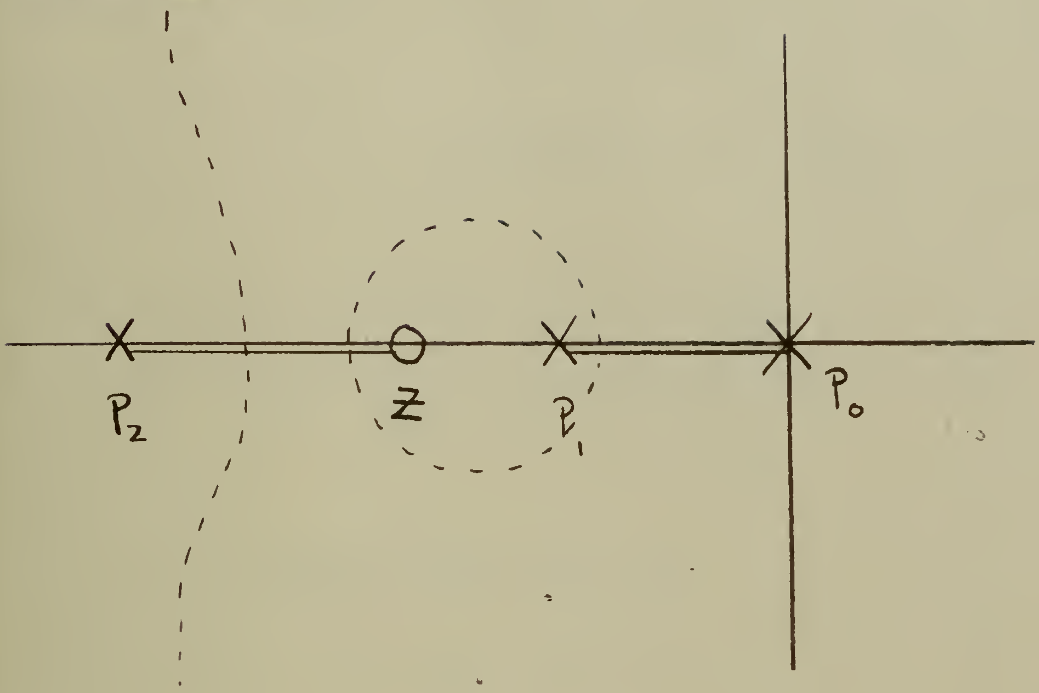


This illustrates limiting case for breakaway point.

Thus the results indicate that the expected semi circles and circles will closely approximate the root locus under study. However, if α is greater than (0.5) P/2, this is no longer true. As a matter of fact, there is a maximum deviation of 5% in letting α increase from (0.1) P/2 to (0.5) P/2. The following figures indicate clearly these deviations. The data to plot these curves was obtained by utilizing the digital computers using the program specified in APPENDIX C.

To further prove that this type of analysis will yield the correct value for the breakaway points, consider the following examples.

EXAMPLE ONE



A) For breakaway point between P_0 & P_1

$$K = \frac{B(P_1 - B)(P_2 - B)}{(Z - B)} = \frac{B^3 - B^2(P_1 + P_2) + P_1 P_2 B}{(Z - B)}$$

$$\frac{dK}{dB} = \frac{(Z - B)\left[3B^2 - 2B(P_1 + P_2) + P_1 P_2\right] + \left[B^3 - 2B^2(P_1 + P_2) + P_1 P_2 B\right]}{(Z - B)^2}$$

but $\frac{dK}{dB} = 0$

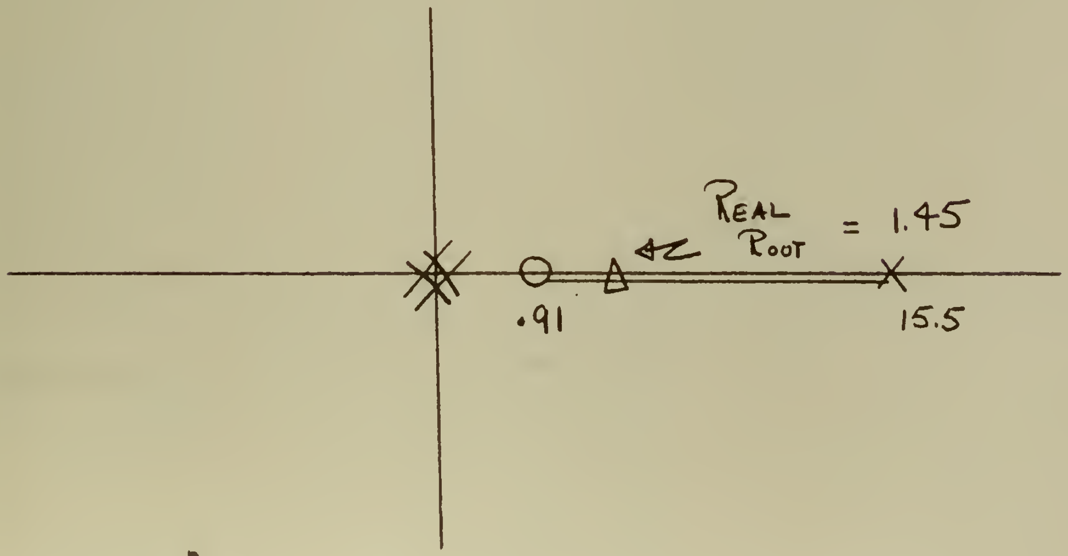
$$\therefore \quad 0 = -2B^3 + B^2\left[3z + P_1 + P_2\right] + B\left[-2z\left(P_1 + P_2\right)\right] + P_1 P_2 z = \frac{dK}{dB}$$

$$1 + \frac{B\left[-2z\left(P_1 + P_2\right)\right] + P_1 P_2 z}{B^2\left[-2B + 3z + P_1 + P_2\right]} = 0 \qquad (I\text{-}2)$$

a) If positive numbers are used

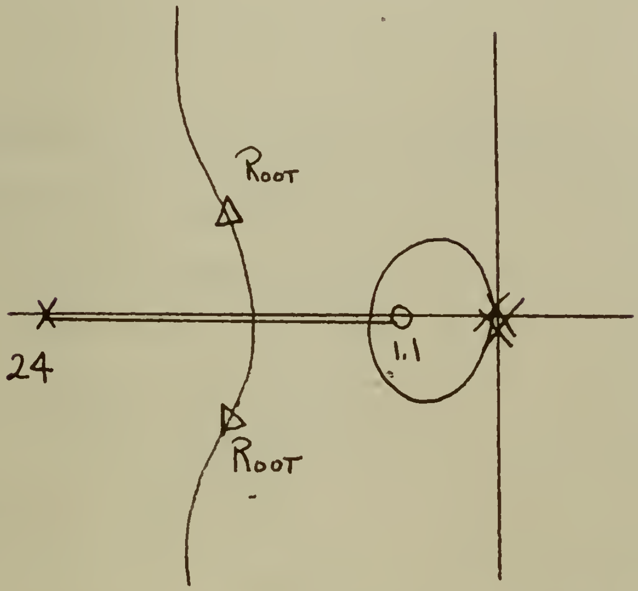
$\therefore P_1 = 2.0, \quad P_2 = 20, \quad z = 3$

$$1 + \frac{66\left[B - .91\right]}{B^2\left[B - 15.5\right]} = 0$$



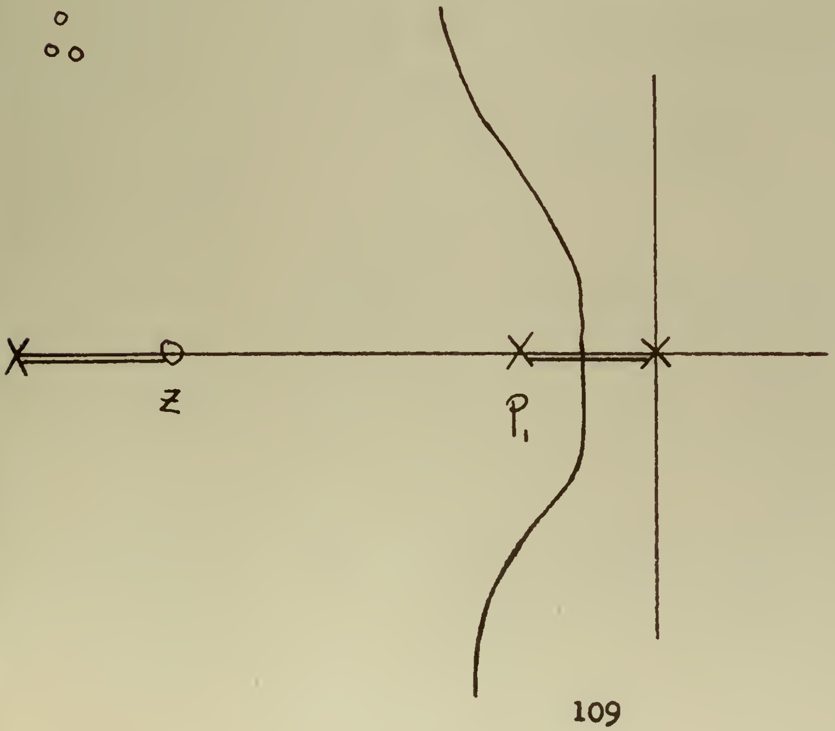
To illustrate further, let $P_1 = -2 \quad P_2 = -20 \quad Z = -10$

$$\therefore \quad 1 + \frac{180\,(B + 1.1)}{B^2\,(B + 24)} = 0$$



Complex roots are obtained in this case which means breakaway point between P_2 and Z does not exist.

$\therefore$



b)

$P_1 = -2$

$P_2 = -20$

$z = -3$

$$1 + \frac{66\,[B + .91]}{B^2\,[B + 15.5]} = 0$$

Therefore the same answer is obtained only with a reversal of sign which indicates the breakaway point will lie between P_o and P_1 in the LHP.

B) for breakaway between P_o and P_1

$$K = \frac{B(p_2 - B)(B - p_1)}{(B - z)}$$

$$= \frac{-B^3 + B^2(p_2 - p_1) - p_2 p_1 B}{(B - z)}$$

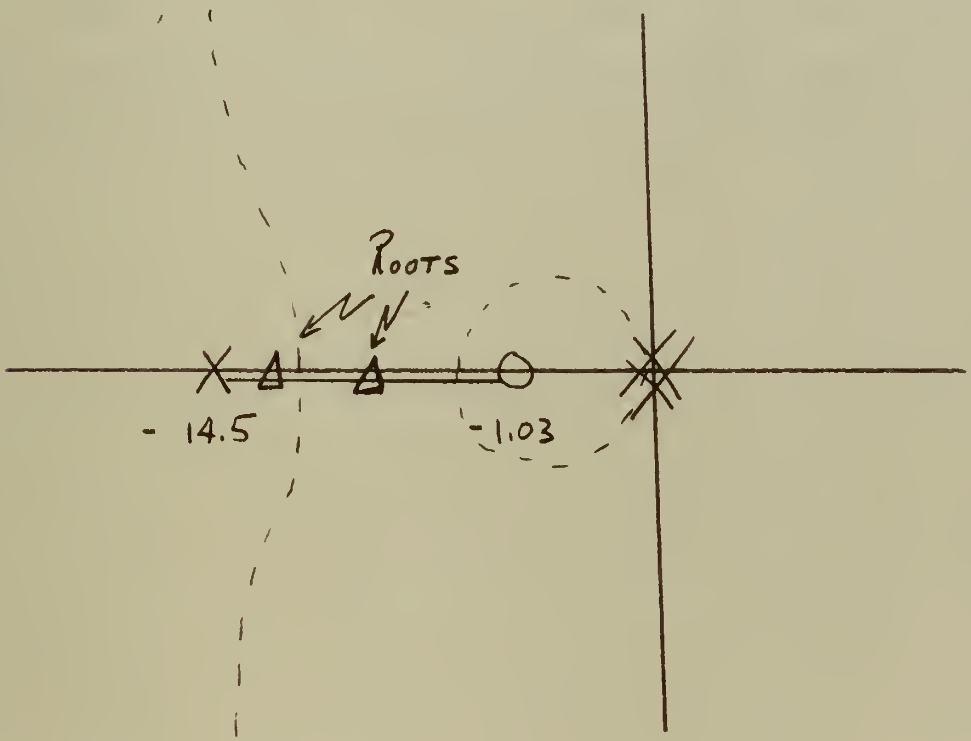
$$\frac{dK}{dB} = \frac{(B - z)\left[-3B^2 + 2B(p_2 - p_1) - p_2 p_1\right] - \left[B^2 + B^2(p_2 - p_1) - p_1 p_2 B\right]}{(B - z)^2}$$

$$0 = -2B^3 + B^2\left[3z + (p_2 - p_1)\right] + B\left[-2z(p_2 - p_1\right] + p_2 p_1 z$$

$$0 = 1 + \frac{B\left[-2z(p_2 - p_1)\right] + p_2 p_1 z}{B^2\left[-2B + (3z + p_2 - p_1)\right]} \qquad (\text{II-2})$$

Comparing this with (I-2), the only difference is in the sign of the summation of P_2 and P_1. Therefore the case where $P_1 = -2$, $P_2 = -20$ & $Z = -3$

$$1 + \frac{54(B + 1.03)}{B^2(B + 14.5)}$$



Root occur at - 4 and - 9.

Thus all three breakaway points for this problem have been obtained

From the previous examples, it has been proven that a breakaway point or several breakaway points can be found by solving the expression of gain along the real axis in terms of B (the breakaway point), differentiating it, setting it equal to zero and solving for the real roots of this resulting equation. If the breakaway points exist they will show up as real roots; if they don't exist, all roots in that section will be complex.

It should be pointed out that each segment on the root locus that may contain a breakaway point must be analyzed separately since the reversal of the vectors may alter the expression of another section.

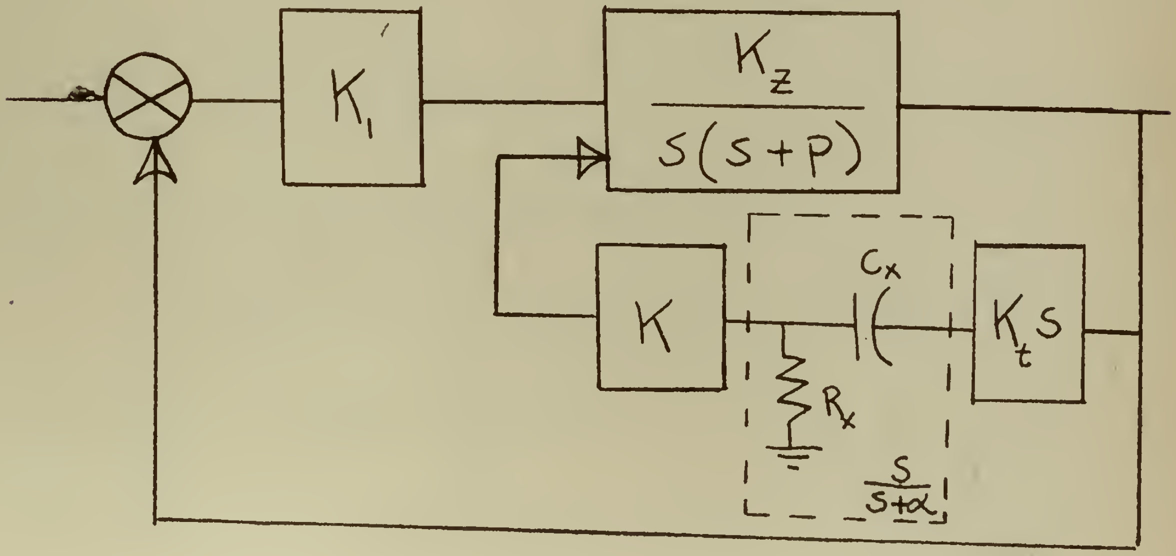
APPENDIX B

ANALOGUE COMPUTER SIMULATION OF COMPENSATION PROBLEM: TO OBTAIN TRANSIENT RESPONSE, CHARACTERISTICS

In the following discussion, the methods employed in simulating the actual system utilizing the analogue computer will be illustrated; a comparison between the transient response obtained with this set up and one obtained mathematically will be made (to illustrate the validity of the answers obtained using this set up), and finally the results of a study of the transient response characteristics for the compensated system varying the initial damping ratio (ζ_{i}^{o}) and, the compensator pole, (α) will be presented.

Referring to the system of interest it will now be shown how an analogue computer can be utilized to illustrate its transient characteristics. The method to be employed is frequently called the "transfer function" method of simulation.

Taking:

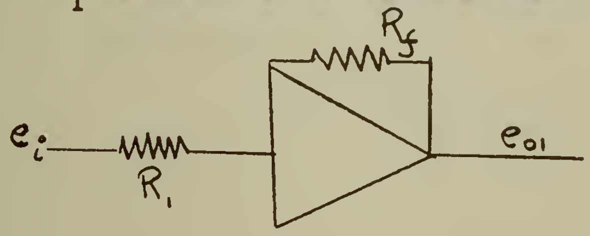


Where $$K_a \triangleq K K_t$$

and $$\alpha = \frac{1}{R_x C_x}$$

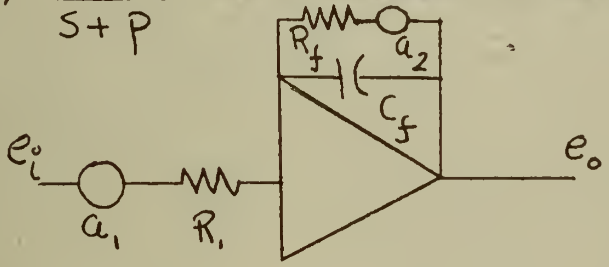
Utilizing the transfer function method, it will be noted:

1) . K_1 can be obtained using a summer whereby



$$e_{o1} = -\frac{R_f}{R_1} e_i$$
$$= -K_1 e_i$$

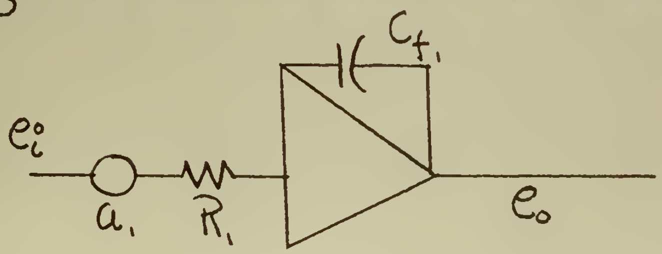
2) $\frac{K_z}{S+P}$ can be obtained from the following



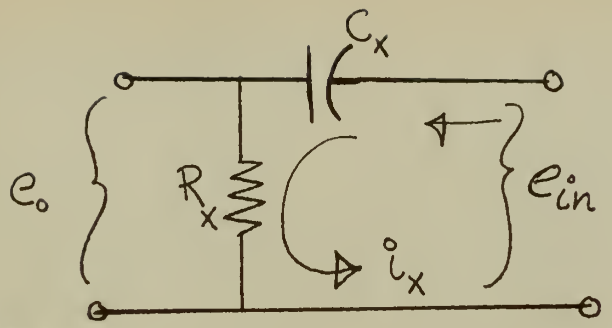
since

$$e_o = \frac{-\dfrac{a_1}{R_1 C_f}}{p + \dfrac{a_2}{R_f C_f}} e_i = \frac{-K_z e_i}{S+P}$$

3) $\frac{1}{S}$ can be obtained using just an integrator of the form



$$e_o = -\frac{1}{P} \frac{a_1}{R_1 C_f} e_i$$
$$= \frac{1}{S} e_i$$



4) Finally since

$$e_{in} = R_x i_x + \frac{1}{C_x} D^{-1} i_x$$

$$= \left[\frac{s C_x R_x + 1}{s C_x}\right] i_x$$

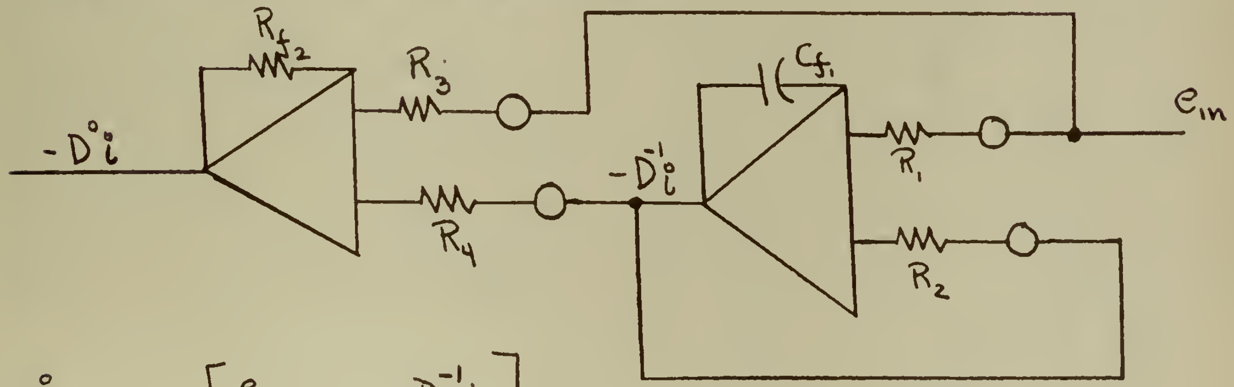
but $e_o = R_x i_x$

therefore

$$e_o = \frac{R_x s C_x}{s C_x R_x + 1} e_{in} = \frac{s}{s + \frac{1}{R_x C_x}} e_{in}$$

$$= \frac{s}{s+\alpha} e_{in}$$

This can be obtained using the following



$$-D^0 i_x = -\left[\frac{e_{in}}{R_1} - \alpha D^{-1} i_x\right]$$

AND $$-D^{-1} i_x = \frac{1}{P}\left[\frac{e_{in}}{R_x} - \alpha D^{-1} i_x\right]$$

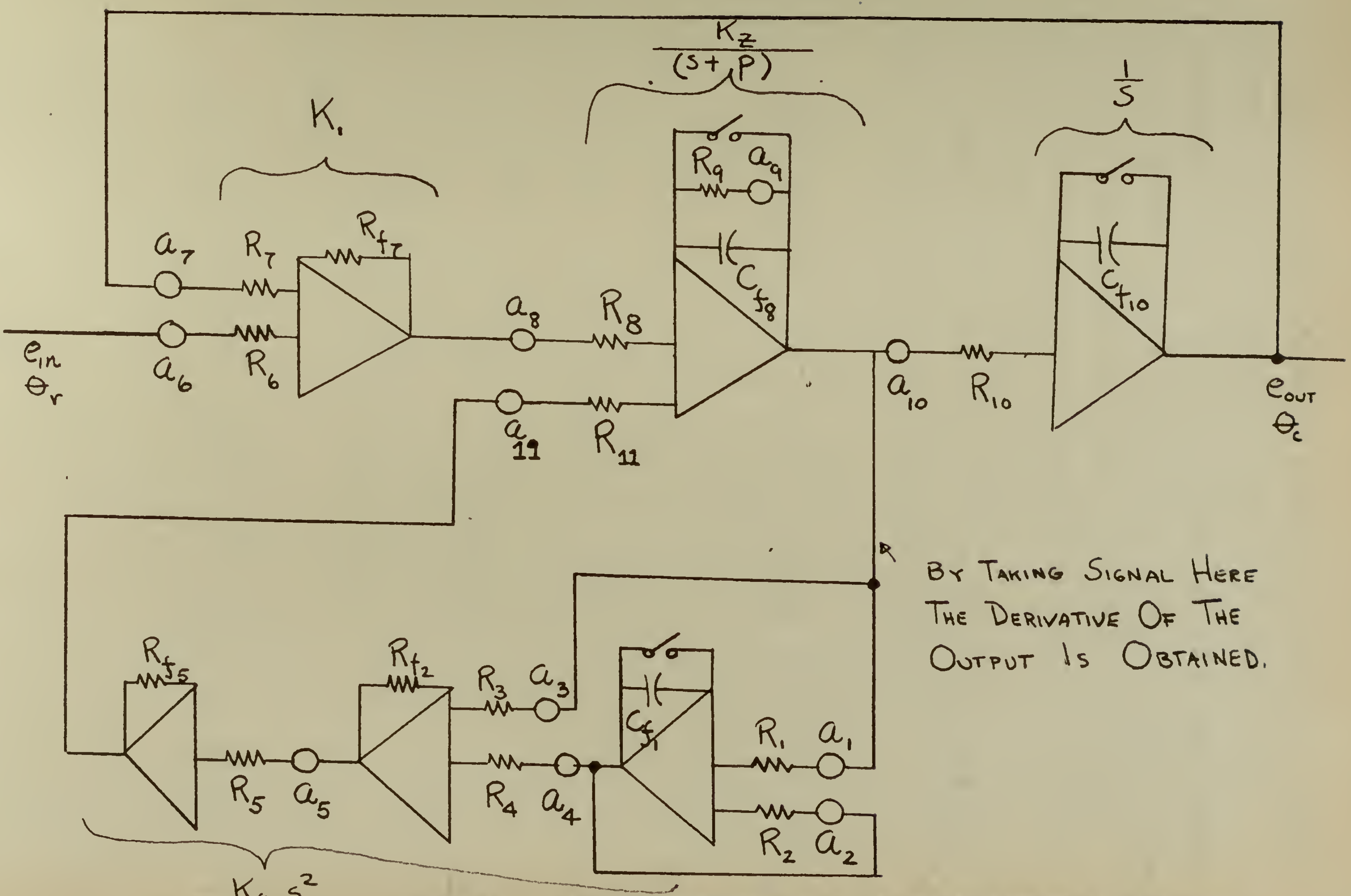
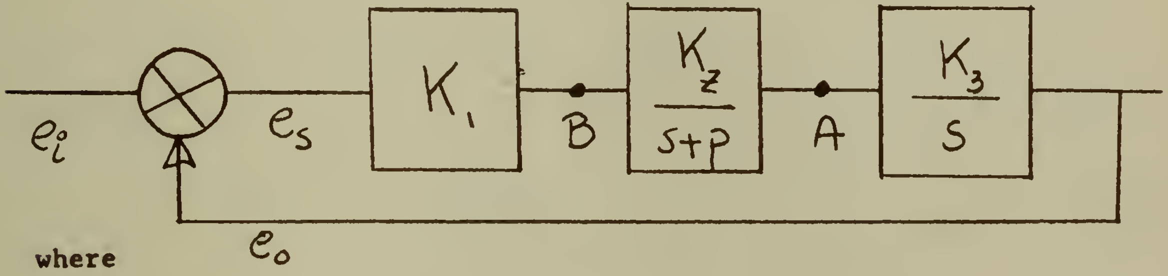


Figure B-1. ANALOGUE COMPUTER SIMULATION OF APPROXIMATE ACCELERATION FEEDBACK OF SECOND ORDER SYSTEM

Using this setup, consider the possibility of obtaining a general expression for the differential equation of the uncompensated and compensated systems in terms of the parameters of the analogue computer (i.e., potentiometer setting, as well as resistance and capacitor values). If this can be accomplished, any initial second order system can be readily simulated and the resultant differential equation of the compensated system can be readily obtained.

To achieve this end, refer back to the actual analogue computer setup. It will be noted for the uncompensated system that



where

$$e_o = \frac{1}{s}\,\frac{a_{10}}{R_{10}C_{10}}\,A$$

$$A = \frac{R_9 a_8}{R_8}\left(\frac{1}{a_9 + R_9 C_8 s}\right)$$

$$B = \frac{a_7 R_{f7}}{R_7}\left(e_i - e_o\right)$$

therefore

$$e_o = \frac{1}{s}\,\frac{a_{10} R_9 a_8 a_7 R_{f7}}{R_{10} C_{10} R_8 R_7}\left(e_i - e_o\right)\left[\frac{1}{a_9 + R_9 C_8 s}\right]$$

so that

$$\begin{aligned} & R_7 R_8 R_{10} R_9 C_8 C_{10} s^2 e_o \\ & \qquad + R_7 R_8 R_{10} C_{10} a_9 s e_o \\ & \qquad\qquad + a_7 a_8 a_{10} R_{f7} R_9 e_o = \end{aligned}$$

$$\ddot{e} + \frac{a_9}{R_9 C_8} \dot{e}_o + \frac{a_7 a_8 a_{10} R_{f7}}{R_7 R_8 R_{10} C_{10} C_8}$$

which corresponds to

$$\ddot{e} + 2\zeta \omega_n \dot{e}_o + \omega_n^2$$

Therefore

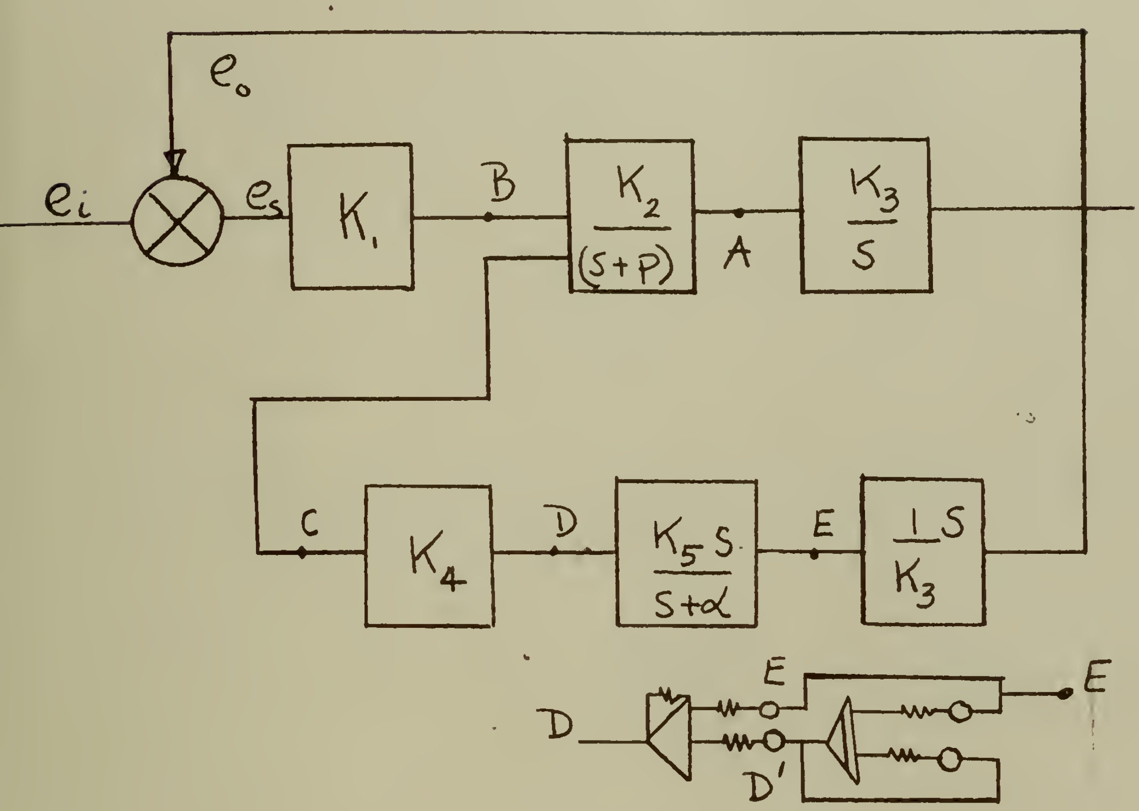
$$2\zeta \omega_n = \frac{a_9}{R_9 C_8}$$

$$\omega_n^2 = \frac{a_7 a_8 a_{10} R_{f7}}{R_7 R_8 R_{10} C_{10} C_8}$$

This then is the desired general expression for the uncompensated system. Now given the equation of the initial system, the appropriate values for the computer parameters can be selected to represent this equation and allow the computer's amplifiers to remain in the unsaturated region. This will not only allow proper simulation, but permits accurate results to be obtained.

Performing this same type of analysis on the compensated system, the following results:

For the compensated system:



1) $$e_o = \frac{1}{S} \frac{a_{10}}{R_{10} C_{10}} A$$

2) $$A = \frac{R_9}{a_9 + R_9 C_8 S} \left[\frac{a_8}{R_8} B - \frac{a_{11}}{R_{11}} C \right]$$

3) $$B = \frac{a_7 R_{f_7}}{R_7} \left(e_i - e_o \right)$$

4) $$C = \frac{a_5 R_{f_5} D}{R_5}$$

5) $$D = -\frac{a_4 R_{f_2} D'}{R_4} + \frac{a_3 R_{f_2} E}{R_3}$$

6) $$D' = \frac{1}{s}\frac{a_1 E}{R_1 C_1} - \frac{1}{s}\frac{a_2 D'}{R_2 C_1}$$

7) $$E = s\frac{R_{10} C_{10}}{a_{10}} e_o$$

Then solving for e_o in terms of the computer parameters using the above seven equations, the following results are obtained

From (6) $$D'\left(1 + \frac{1}{s}\frac{a_2}{R_2 C_1}\right) = \frac{1}{s}\frac{a_1}{R_1 C_1} E$$

OR

$$D' = \frac{a_1 R_2 \quad E}{R_1 \ (s R_2 C_1 + a_2)}$$

From (5)

$$D = \left[\frac{a_3 R_{f_2}}{R_3} - \frac{a_4 R_{f_2} a_1 R_2}{R_4 R_1 \left[s R_2 C_1 + a_2\right]}\right] E$$

but from (4)

$$C = \frac{a_5 R_{f5} D}{R_5} = \frac{a_5 R_{f5} E}{R_5}\left[\frac{a_3 R_{f2}}{R_3} - \frac{a_4 R_{f2} a_1 R_2}{R_1 R_4 (s R_2 C_1 + a_2)}\right]$$

and from (1) and (2)

$$e_o = \frac{1}{s}\,\frac{a_{10} R_9}{R_{10} C_{10} (a_9 + R_9 C_8 s)}\left[\frac{a_8 B}{R_8} - \frac{a_{11} C}{R_{11}}\right]$$

Therefore by substituting in the expressions for B and C those containing e_o, separating all coefficients of $\dddot{e}_o$, $\ddot{e}_o$, $\dot{e}_o$ and e_o to one side of the equation, the following result is obtained

$$\dddot{e}_o \left(R_9 C_8 C_1 R_2\right)$$

$$+ \ddot{e}_o \left(C_1 R_2 a_9 + R_9 C_8 a_2 + \frac{R_9 a_{11} a_5 R_{f5} R_2 C_1 R_{f2} a_3}{R_{11} R_3 R_5}\right)$$

$$+ \dot{e}_o \left(a_9 a_2 + \frac{a_{10} R_9 a_7 a_8 R_{f7} C_1 R_2}{R_{10} C_{10} R_7 R_8}\right)$$

$$+ e_o \left(\frac{a_{10} R_9 a_7 a_8 R_{f7} a_2}{C_{10} R_{10} R_7 R_8}\right)$$

Letting $\dddot{e}_o$ have a coefficient of unity yields

$$\dddot{e}_o + \ddot{e}_o\left[\frac{C_1R_2a_9+R_9C_8a_2}{R_9C_8C_1R_2}+\frac{a_{11}a_5R_{f5}R_{f2}a_3}{C_8R_{11}R_4R_5}\right]$$

$$+\dot{e}_o\left[\frac{a_9a_2}{R_9C_8C_1R_2}+\frac{a_{10}a_7a_8R_{f7}}{R_{10}C_{10}R_7R_8C_8}\right]$$

$$+e_o\left[\frac{a_{10}a_7a_8a_2R_{f7}}{C_{10}R_{10}R_7R_8C_8C_1R_2}\right]$$

Therefore, for this particular system in order for the computer to properly simulate its response its characteristic equation

$$s^3+(K_aK_z+p+\alpha)s^2+(K_1K_z+p\alpha)s+K_1K_z\alpha$$

provides

$$K_aK_z+p+\alpha = \frac{a_{11}a_5R_{f5}R_{f2}a_3}{C_8R_{11}R_4R_5}+\frac{C_1R_2a_9+R_9C_8a_2}{R_9C_8C_1R_2}$$

$$K_1K_z+p\alpha = \frac{a_9a_2}{R_9C_8C_1R_2}+\frac{a_{10}a_7a_8R_{f7}}{R_{10}C_{10}R_7R_8C_8}$$

$$K_1K_z\alpha = \frac{a_{10}a_7a_8a_2R_{f7}}{C_{10}R_{10}R_7R_8C_8C_1R_2}$$

To consider briefly what must be changed to effectively move (α) the compensator pole or (ρ_i^o) the initial damping ratio

Note

1) If it is desired to change the position of α - change potentiometer settings a_4 & a_2

$$a_4 = \frac{\alpha R_4}{R_{f_2}} \qquad a_2 = \alpha R_2 C_{f_1}$$

2) If it is desired to change $K_1 K_z$ (which will move initial roots up or down to get new ρ_i^o) - change potentiometer settings $a_7\, a_8\, a_{10}$

Since

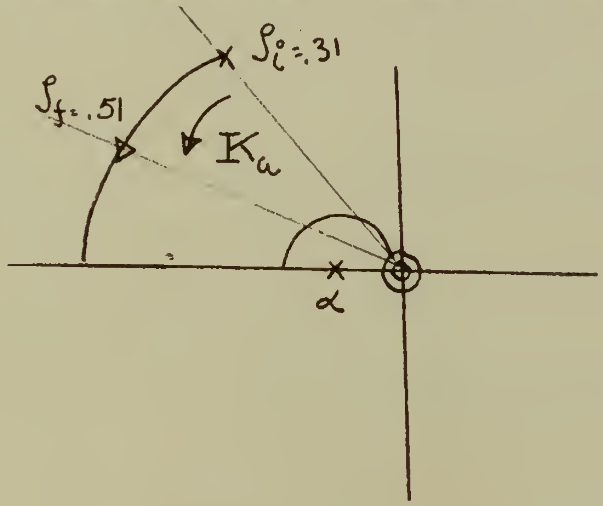
$$K_1 K_z = a_7\, a_8\, a_{10} \frac{R_{f_7}}{R_7 R_8 R_{10} C_{10} C_8}$$

3) If it is desired to change the location of the motor pole "P" - change potentiometer setting a_9

At this point, it is appropriate to illustrate the fact that the results obtained using this setup are well within the degree of accuracy desired and correspond closely to expected values. A mathematical solution will be presented first and then the results obtained using the analogue computer will be checked with it.

Example #1

Consider the case where it is desired to obtain the transient response of a second order system compensated using acceleration feedback. Initially, the roots of the second order system are at S = - 1 ± j 3 which implies an initial ζ = .31. However, it is desired to have the roots of the system at S = - 1.5 ± j 2.8 so that ζ_f = .51. If the pole p = 2.0 and α is selected to be 0.1, what will the transient response be?



Initially the characteristic equation is:

$$s^2 + ps + K_1 K_2 = 0 = s^2 + 2\zeta\omega_n s + \omega_n^2$$

where

$$2\zeta\omega_n = 2 \qquad\qquad \omega_n^2 = 10.2$$

But to have roots at desired locations using acceleration feedback, the appropriate gain at the roots can be obtained from the root locus discussed on Appendix A.

It is found that

Gain at complex roots = .936

With this value of gain the third root of the compensated system can be found. This root is real and equal to 0.101.

Multiplying these roots will yield the new characteristic equation from which the system function can be found and the transient response calculated.

That is $(s + 1.5 + j2.8)(s + 1.5 - j2.8)(s + .101)$

equals $s^3 + 3.10\,s^2 + 10.4\,s + 1.02$

This checks with what is expected since

$$K_1 K_2 \alpha = 1.02 \qquad K_1 K_2 + p\alpha = 10.4$$

$$K_1 K_2 = 10.2 \qquad K_1 K_2 = 10.4 - .2$$

$$= 10.2$$

This agreed with coefficients used in the characteristic equation.

The system function is then noted to be

$$\frac{\theta_c}{\theta_r} = \frac{K_1 K_2 (s+\alpha)}{s^3 + (K_a K_2 + p + \alpha)s^2 + (K_1 K_2 + p\alpha)s + K_1 K_2 \alpha}$$

$$= \frac{10.2\,(s + .100)}{(s+1.5+j2.8)(s+1.5-j2.8)(s+.101)}$$

If $\theta_r = 1/s$ (that is, a step function) the transient response can be obtained using the residue theorem. This results in the following:

$$\theta_c = 1 + 1.146\,e^{-1.5t}\cos(2.8t + 151.8^\circ) - .011\,e^{-.101t}$$

Substituting in value for time yields the transient response.

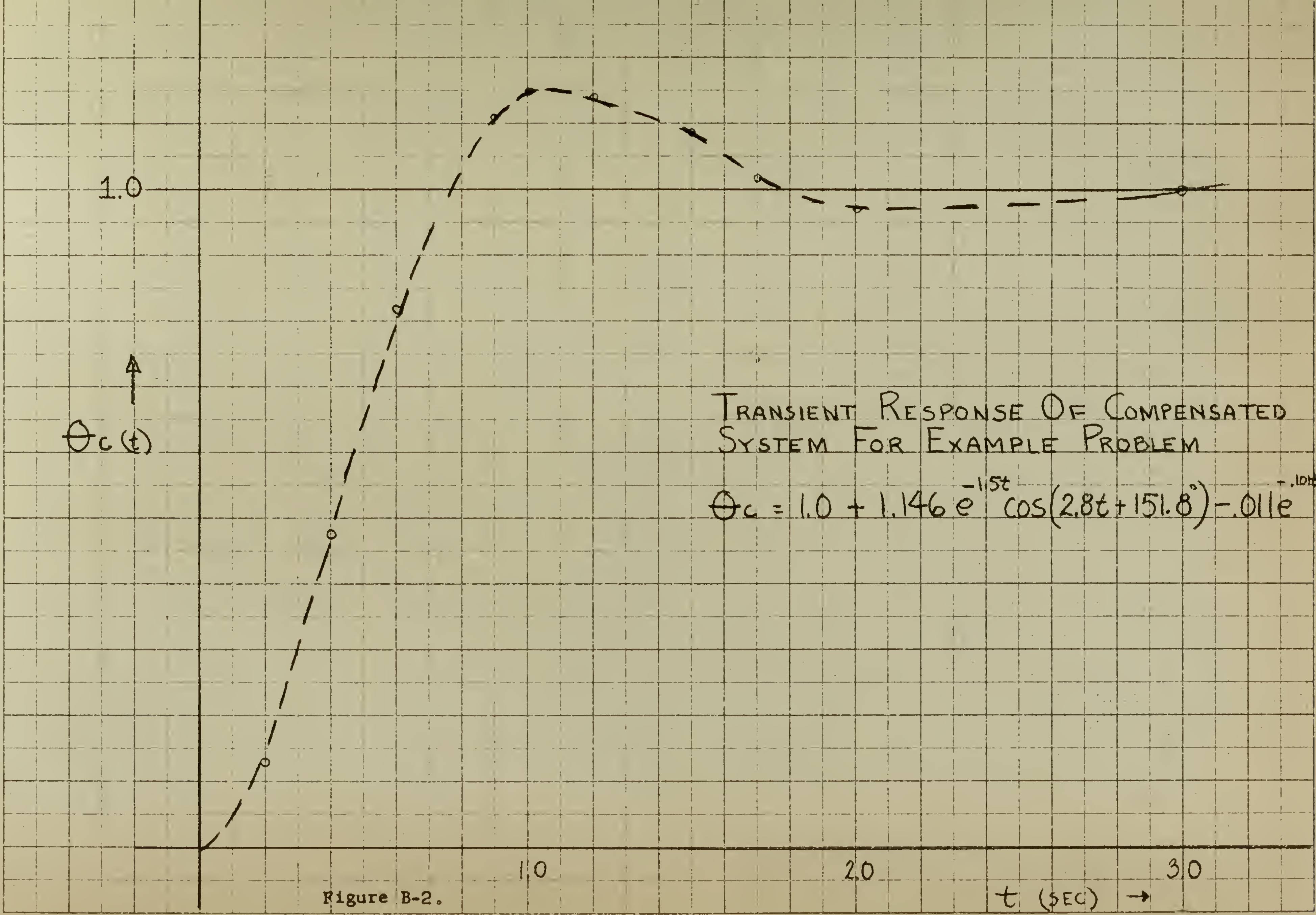

1.0
$\theta_c(t)$
TRANSIENT RESPONSE OF COMPENSATED SYSTEM FOR EXAMPLE PROBLEM
$\theta_c = 1.0 + 1.146 e^{-1.5t} \cos(2.8t + 151.8^\circ) - .011 e^{-.10t}$
1.0
2.0
3.0
t (SEC) →

Figure B-2.

Now consider the solution obtained utilizing the computer. Remembering that the general expression for the uncompensated system yields

$$2\zeta\omega_n = \frac{a_9}{R_9 C_8}$$

$$\omega_n^2 = \frac{a_7 a_8 a_{10} R_{f7}}{R_7 R_8 R_{10} C_{10} C_8}$$

The initial characteristic equation of the system is:

$$S^2 + pS + K_1 K_2 = 0$$

so that

$$2\zeta\omega_n = p = \frac{a_9}{R_9 C_8} = 2$$

and

$$\omega_n^2 = K_1 K_2 = \frac{a_7 a_8 a_{10} R_{f7}}{R_7 R_8 R_{10} C_{10} C_8} = 10.2$$

If $C_8 = .99$ and $R_9 = .10$

$$a_9 = .198$$

if $R_{f7} = 1.0$ $R_7 = .10$ $R_{10} = 1.0$ $R_8 = .10$

$C_8 = .99$ $C_{10} = .96$

$$a_7 a_8 a_{10} = .097$$

and if $a_{10} = 1.00$

$$a_7 = a_8 = \sqrt{.097} = .311$$

This then will simulate the uncompensated system. To further prove that this simulation is achieved, the transient response of the uncompensated system is presented and it will be noted that the $M_{p(t)}$ value corresponds exactly to that value expected for an initial $\zeta_i^o = .31$.

After the system is connected up as it would be when compensated, the value of K_a is then adjusted by variation of the appropriate potentiometer settings until the transient response obtained is exactly the same as the one acquired mathematically. If these potentiometer settings, when substituted in the general expression obtained for the compensated system, yield a characteristic equation equal to the one acquired mathematically, then this analogue computer setup can be considered to properly and accurately simulate the compensation problem.

Since

$A_{10} = 1$
$A_7 = .311$
$A_8 = .311$
$A_2 = .2$
$A_9 = .197$
$A_{11} = .060$

$R_1 = 1.00$
$R_2 = 1.00$
$R_{f1} = 2.00$
$R_3 = .50$
$R_4 = .50$
$R_{f2} = 1.00$
$R_5 = .10$
$R_{f5} = 1.00$
$R_6 = .10$

$R_7 = 1.00$
$R_8 = .10$
$R_9 = .10$
$R_{11} = .10$
$C_{f8} = .99$
$R_{10} = 1.00$
$C_{10} = .96$

are the values required to obtain the desired transient response

a) $$K_a K_z + p + \alpha = \frac{C_1 R_2 a_9 + R_9 C_8 a_2}{R_9 C_8 C_1 R_2} + \frac{a_{11} a_5 R_{f5} R_{f2} a_3}{C_8 R_{11} R_4 R_5}$$

$$= 3.20$$ this checks

b) $$K_1 K_z + p\alpha = \frac{a_9 a_2}{R_9 C_8 C_1 R_2} + \frac{a_{10} a_7 a_8 R_{f7}}{R_{10} C_{10} R_7 R_8 C_8}$$

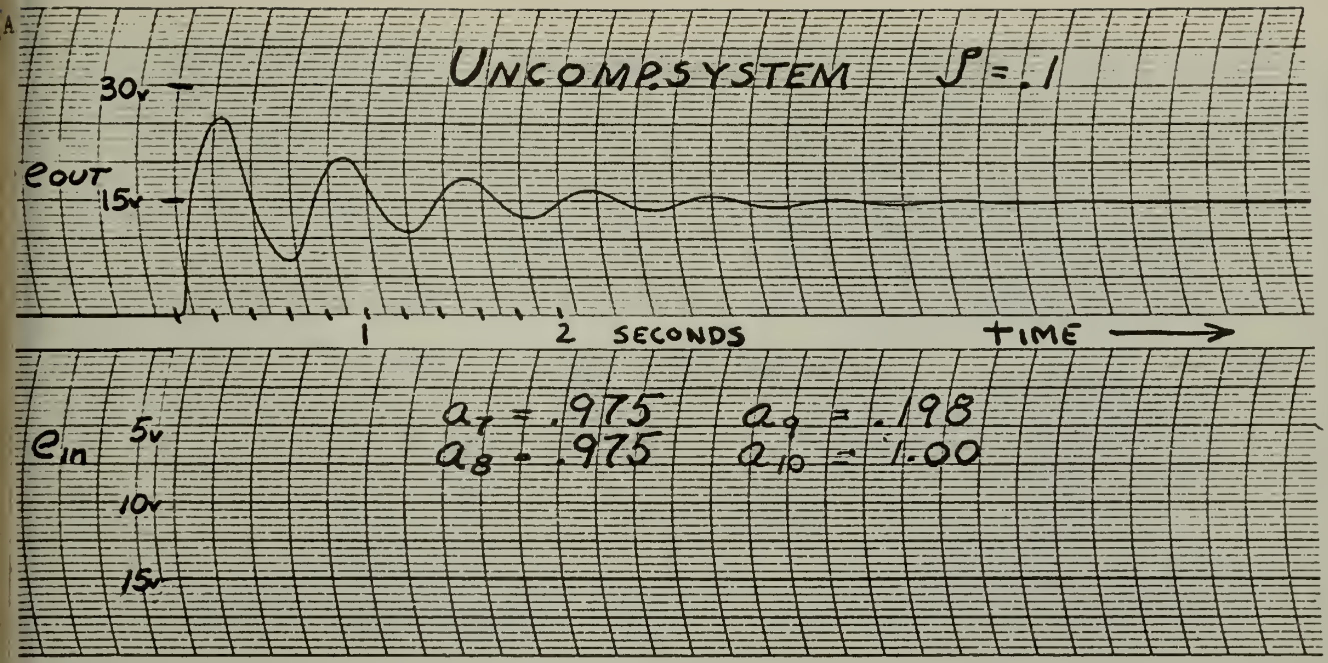
$$= 10.4$$ this checks

c) $$K_1 K_z \alpha = \frac{a_{10} a_7 a_8 R_{f7} a_2}{C_{10} R_{10} R_7 R_8 C_8 C_1 R_2}$$

$$= 1.02$$ this checks

This characteristic equation obtained from the computer setup being equal to the one acquired mathematically constitutes proof enough of the accuracy of this setup.

The results of the variation of α and ρ_i^o are to be presented and it will be noted, that as the gain in the acceleration feedback loop is increased, the transient response is not affected by the presence of the real root. As a matter of fact, if the gain can be made high enough a simple second order response is obtained since the real root is so far out on the negative real axis.



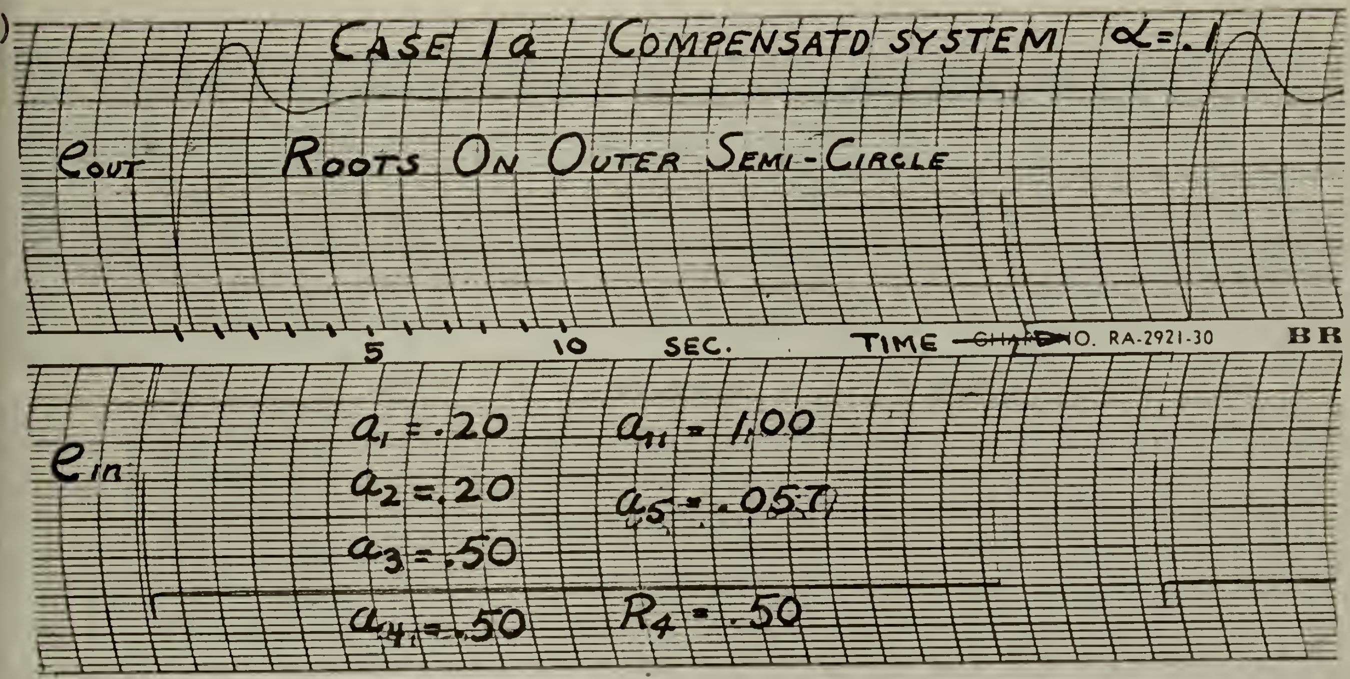
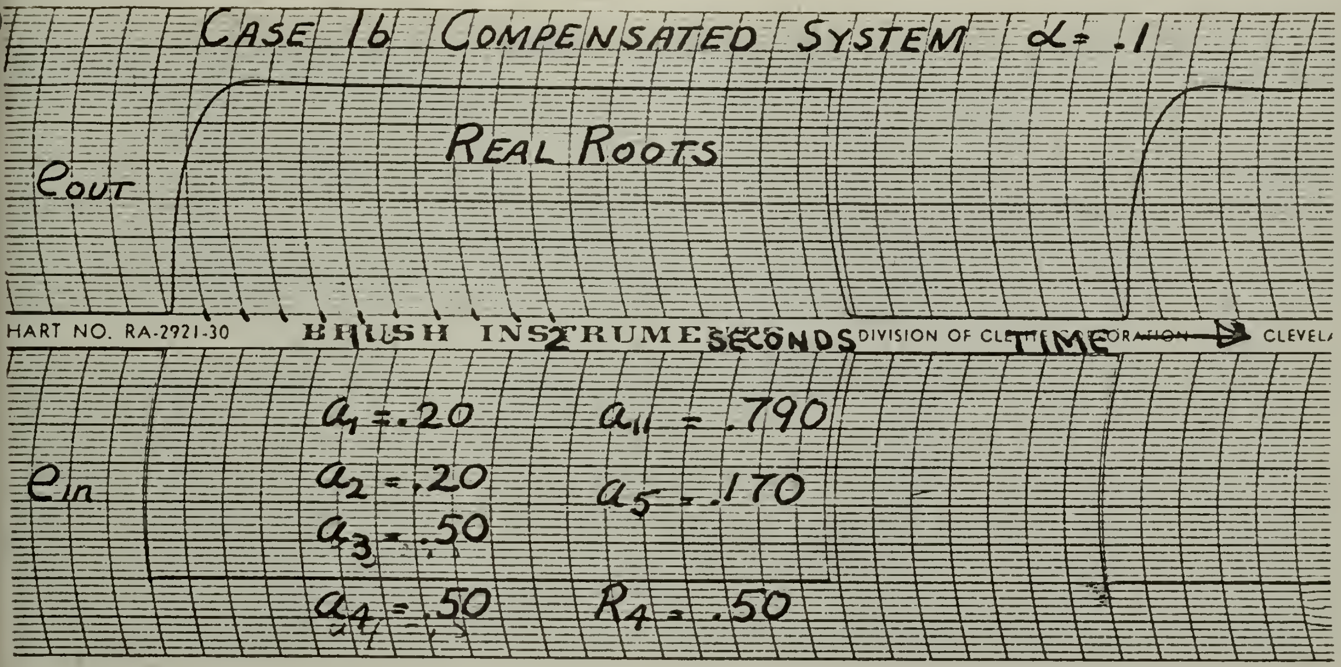
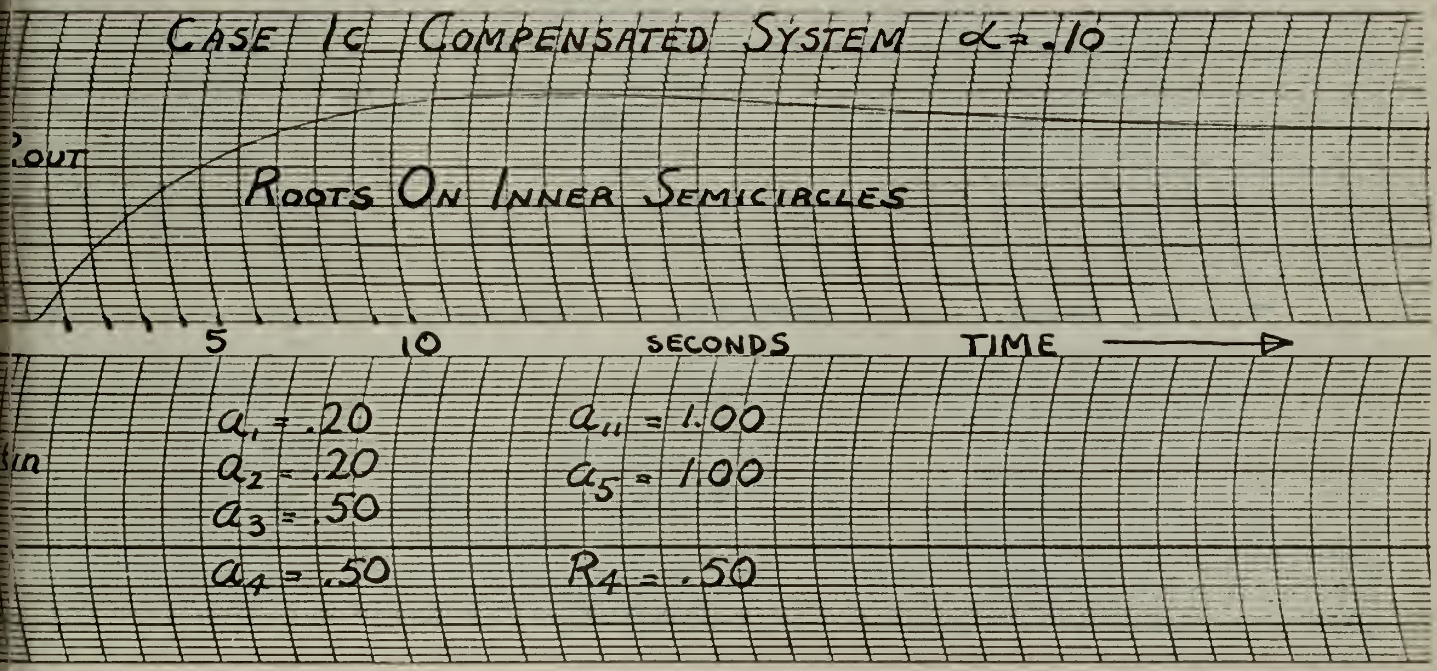


Figure B-4. a - v RESULTS OF COMPENSATION FOR DIFFERENT INITIAL DAMPING RATIOS USING THE ANALOGUE COMPUTER

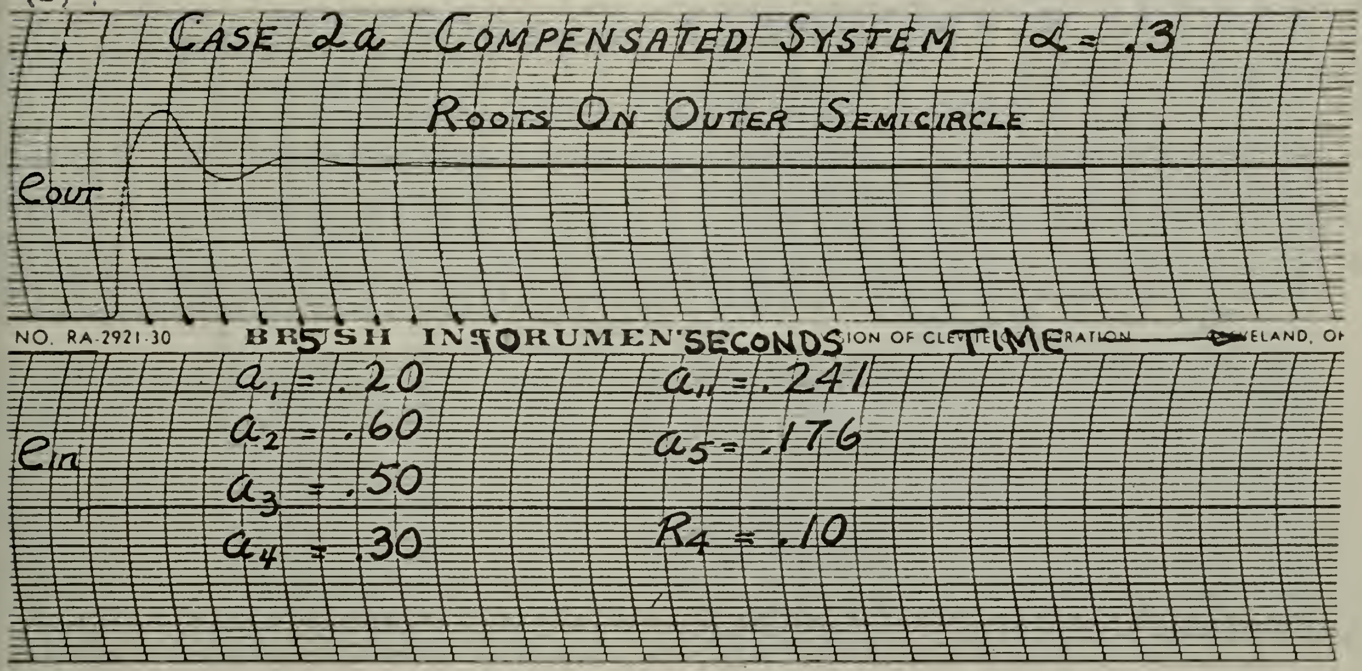
)


CASE 1b COMPENSATED SYSTEM α = .1
REAL ROOTS
e_{OUT}
HART NO. RA-2921-30 BRUSH INSTRUMENTS DIVISION OF CLEVITE CORPORATION CLEVEL
2
SECONDS
TIME
e_{in}
$a_1 = .20$
$a_2 = .20$
$a_3 = .50$
$a_4 = .50$
$a_{11} = .790$
$a_5 = .170$
$R_4 = .50$

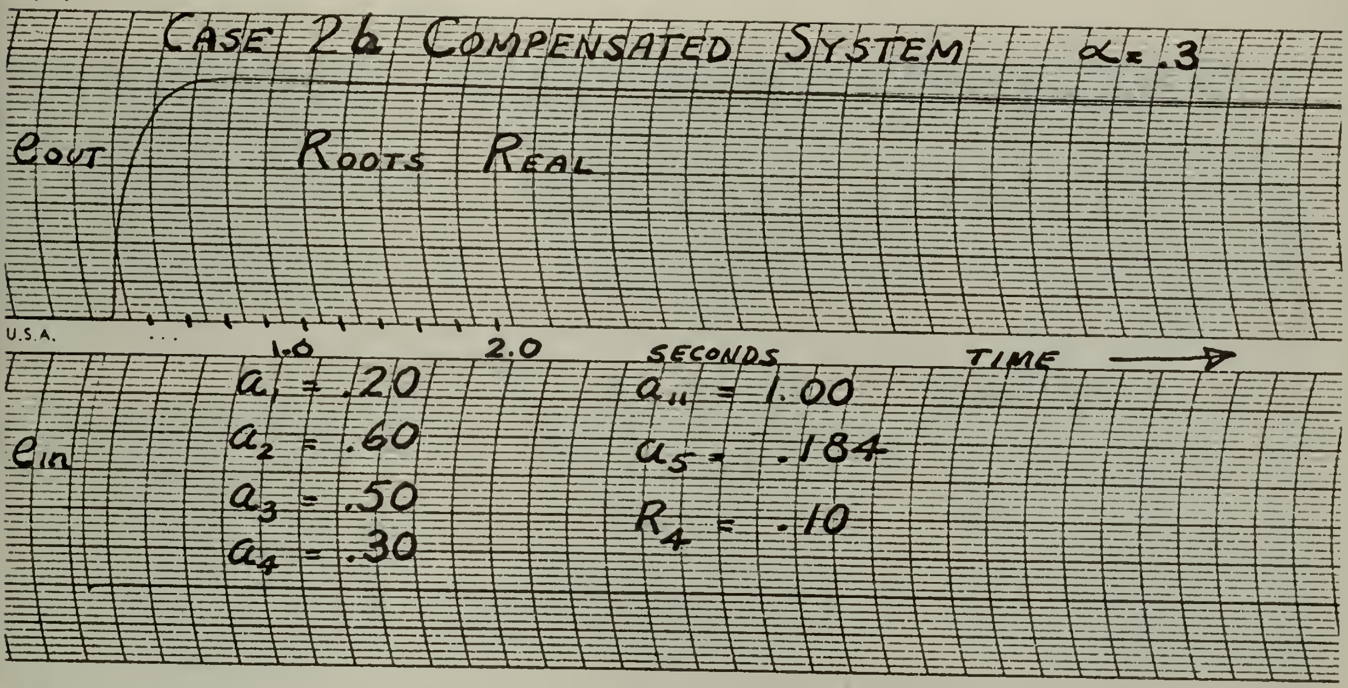
D)


CASE 1c COMPENSATED SYSTEM α = .10
ROOTS ON INNER SEMICIRCLES
e_{OUT}
5
10
SECONDS
TIME
e_{in}
$a_1 = .20$
$a_2 = .20$
$a_3 = .50$
$a_4 = .50$
$a_{11} = 1.00$
$a_5 = 1.00$
$R_4 = .50$

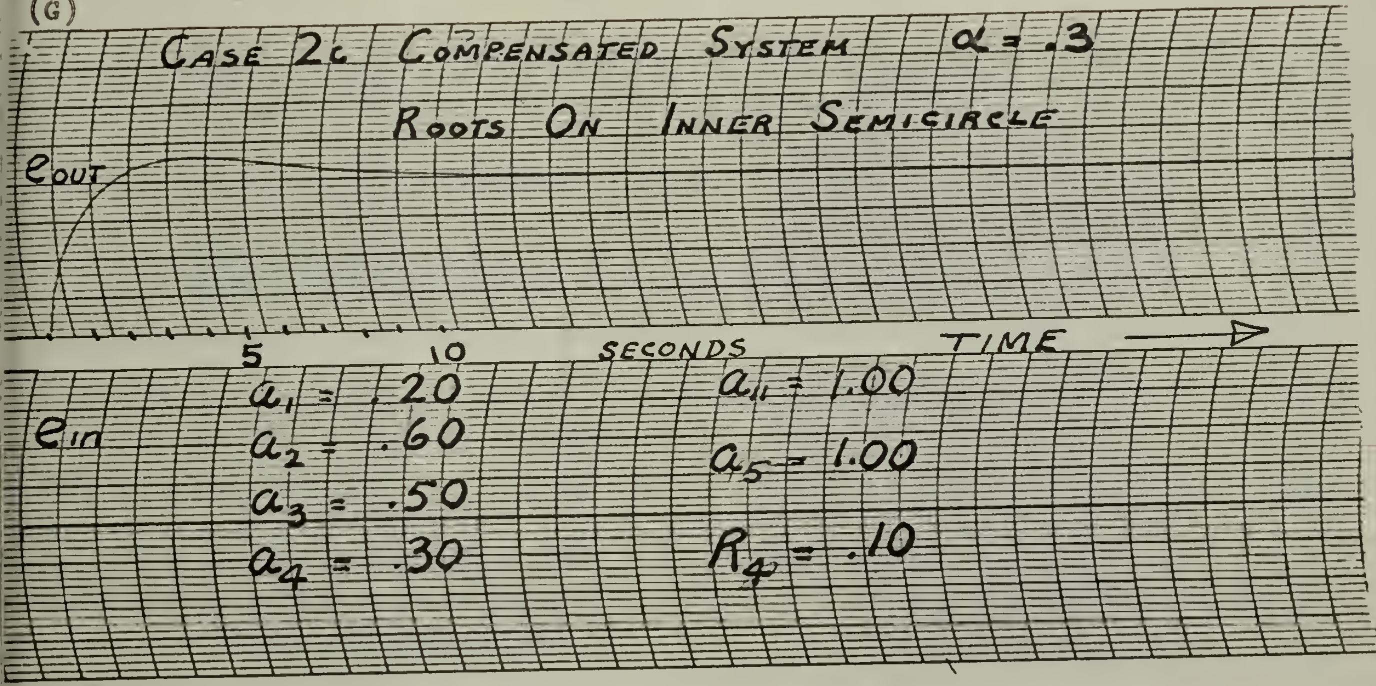
(E)


CASE 2a COMPENSATED SYSTEM α = .3
ROOTS ON OUTER SEMICIRCLE
e_out
NO. RA-2921-30
BRUSH INSTRUMENTS
SECONDS
TIME
e_in
a₁ = .20
a₂ = .60
a₃ = .50
a₄ = .30
a₁₁ = .241
a₅ = .176
R₄ = .10

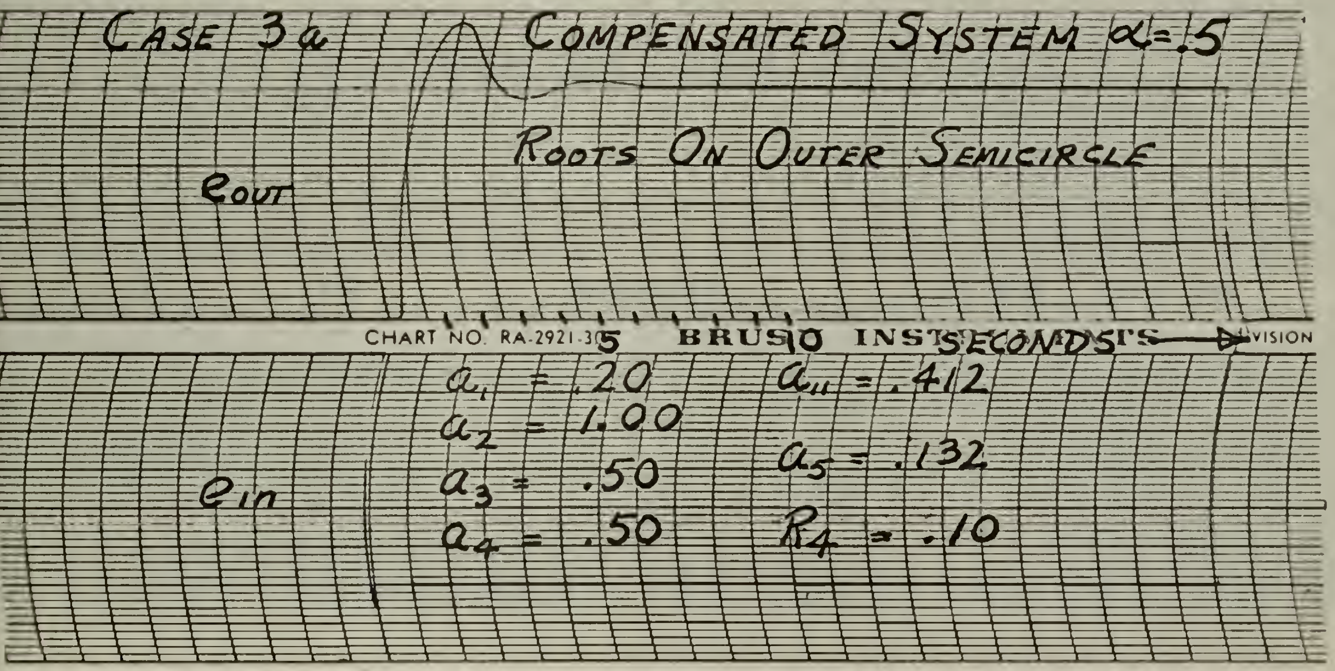
(F)


CASE 2b COMPENSATED SYSTEM α = .3
e_out
ROOTS REAL
U.S.A.
1.0
2.0
SECONDS
TIME
e_in
a₁ = .20
a₂ = .60
a₃ = .50
a₄ = .30
a₁₁ = 1.00
a₅ = .184
R₄ = .10

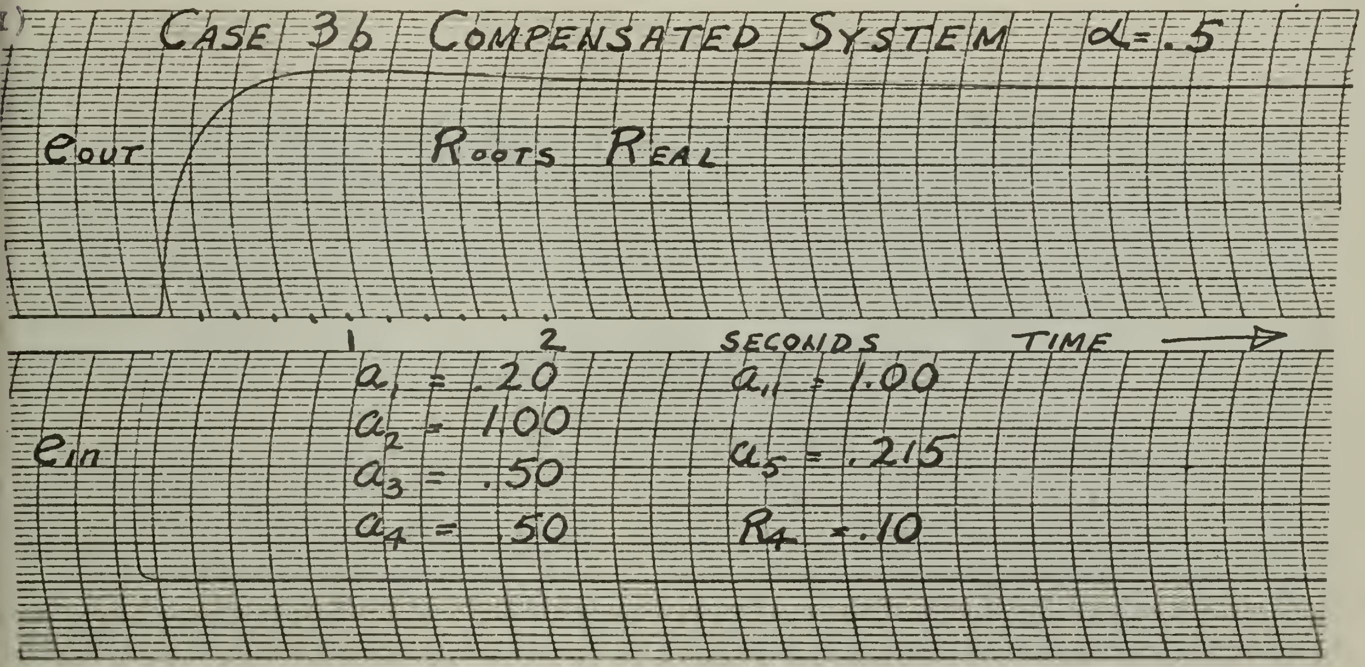
(G)


CASE 2c COMPENSATED SYSTEM α = .3
ROOTS ON INNER SEMICIRCLE
e_OUT
5
10
SECONDS
TIME
e_in
a_1 = .20
a_2 = .60
a_3 = .50
a_4 = .30
a_11 = 1.00
a_5 = 1.00
R_4 = .10

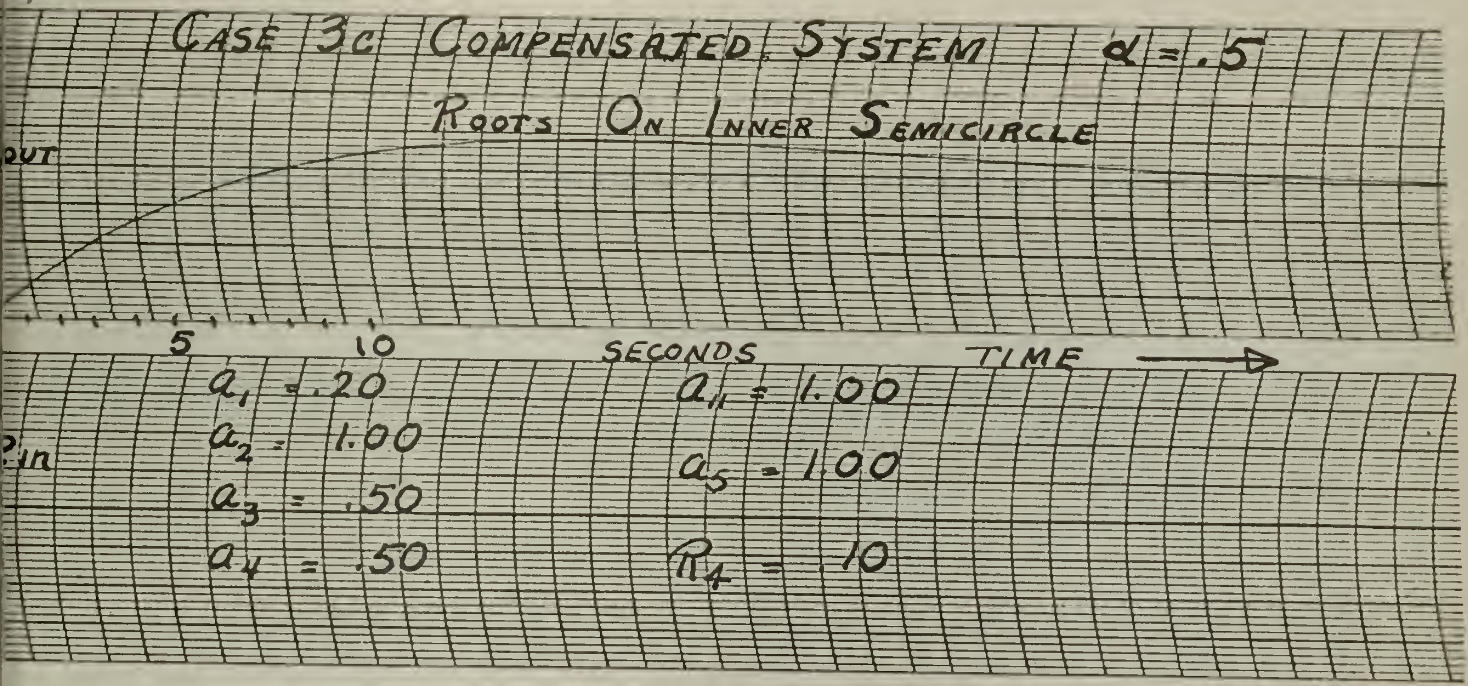
(H)


CASE 3a COMPENSATED SYSTEM α = .5
ROOTS ON OUTER SEMICIRCLE
e_OUT
CHART NO. RA-2921-30
BRUSH INSTRUMENTS
DIVISION
5
10
SECONDS
e_in
a_1 = .20
a_2 = 1.00
a_3 = .50
a_4 = .50
a_11 = .412
a_5 = .132
R_4 = .10

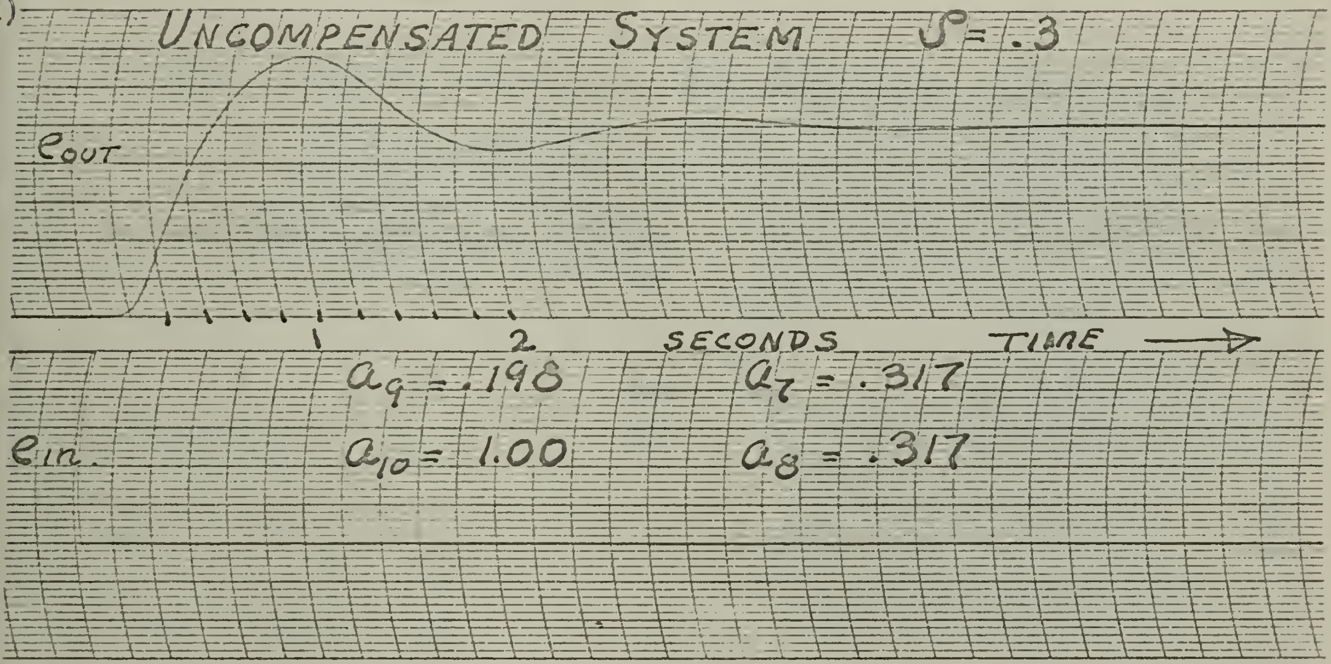
I)


CASE 3b COMPENSATED SYSTEM α = .5
ROOTS REAL
e_{OUT}
e_{in}
1
2
SECONDS
TIME
a_1 = .20
a_2 = 1.00
a_3 = .50
a_4 = .50
a_{11} = 1.00
a_5 = .215
R_4 = .10

J)

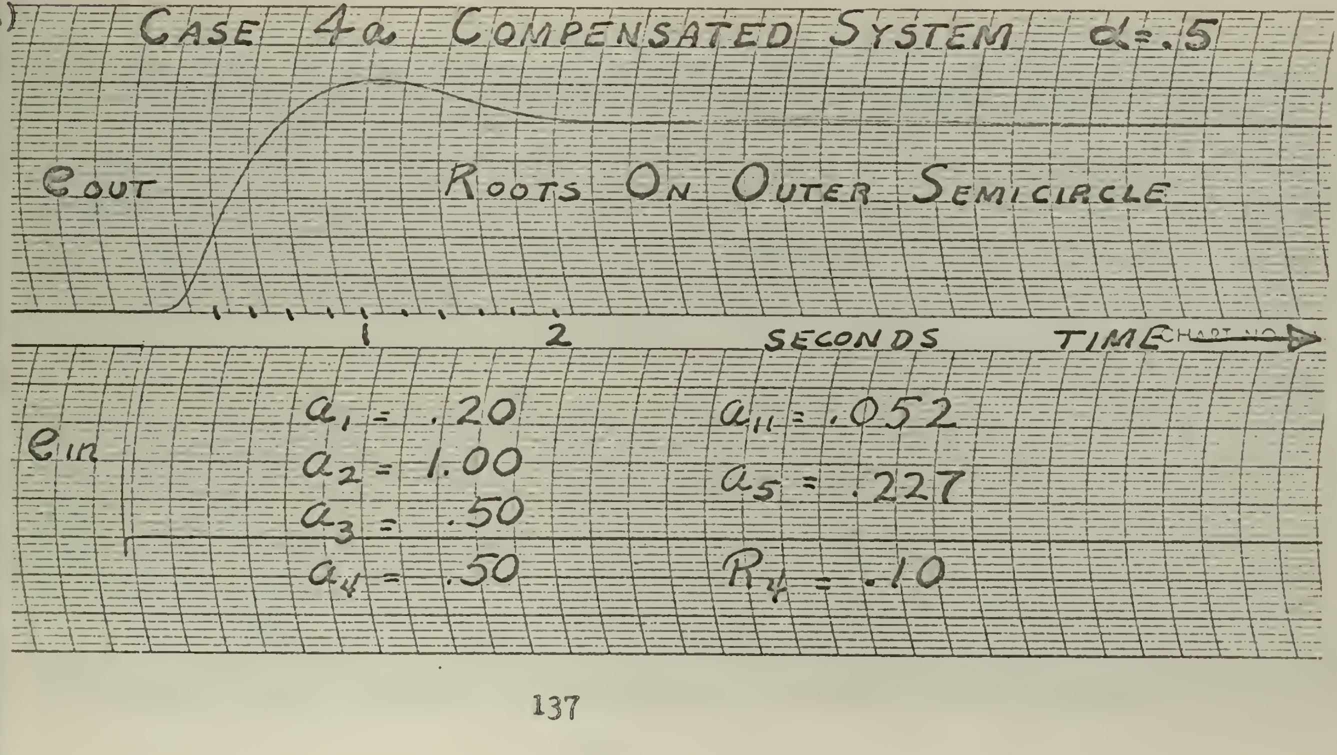

CASE 3c COMPENSATED SYSTEM α = .5
ROOTS ON INNER SEMICIRCLE
OUT
in
5
10
SECONDS
TIME
a_1 = .20
a_2 = 1.00
a_3 = .50
a_4 = .50
a_{11} = 1.00
a_5 = 1.00
R_4 = .10

(K)

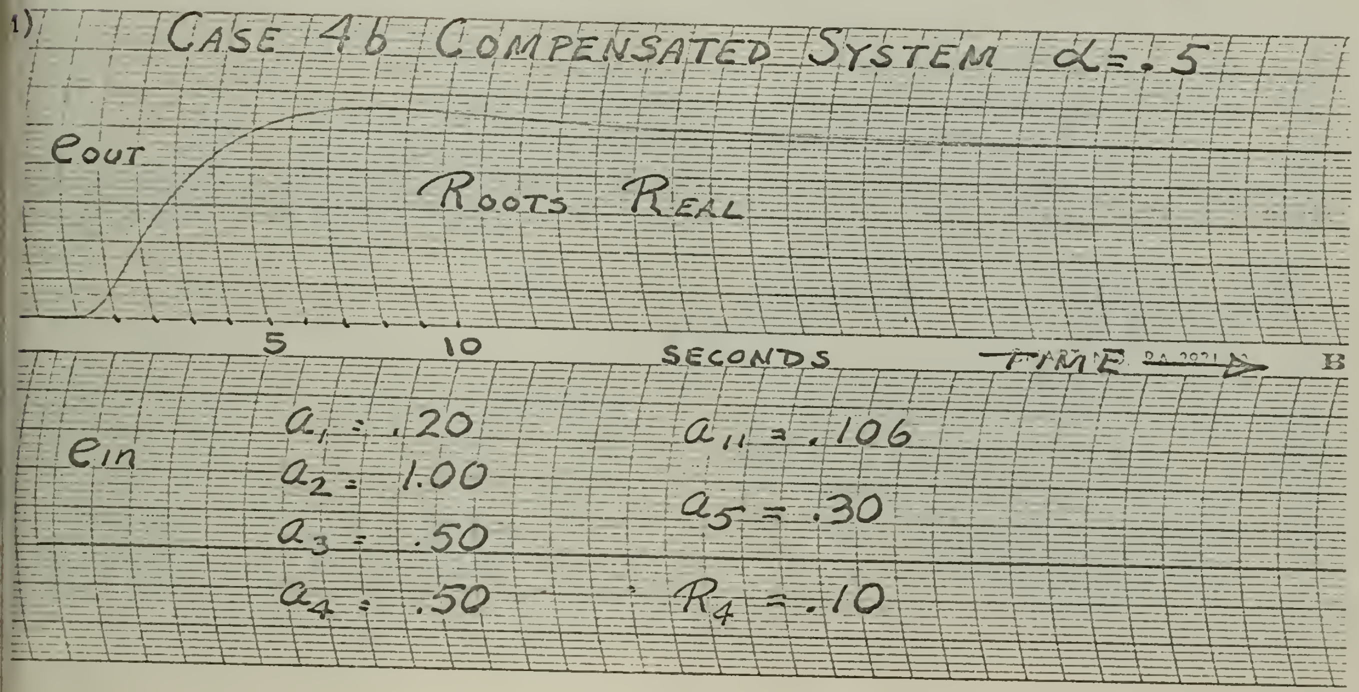


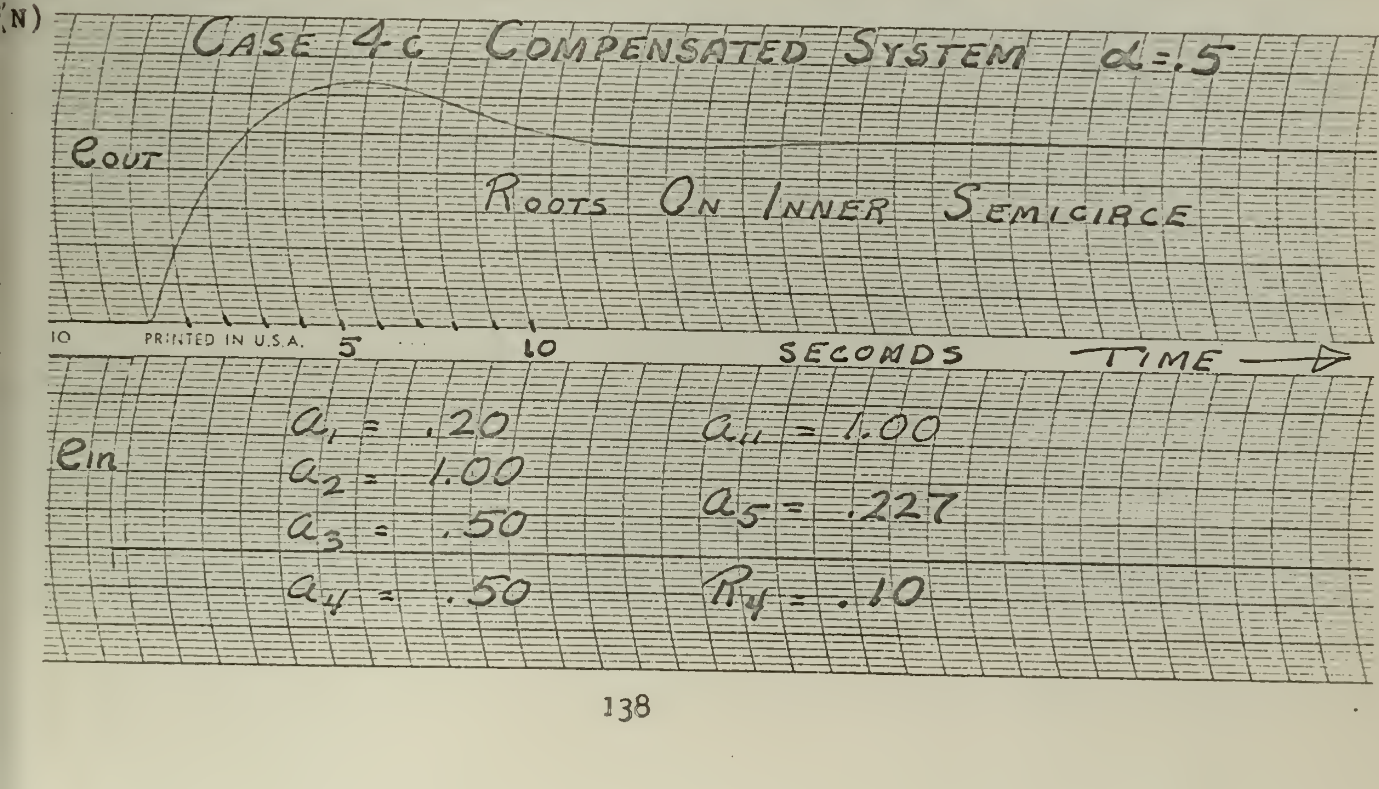
UNCOMPENSATED SYSTEM ζ = .3
e_{out}
1
2
SECONDS
TIME
a_9 = .198
a_7 = .317
e_{in}
a_{10} = 1.00
a_8 = .317

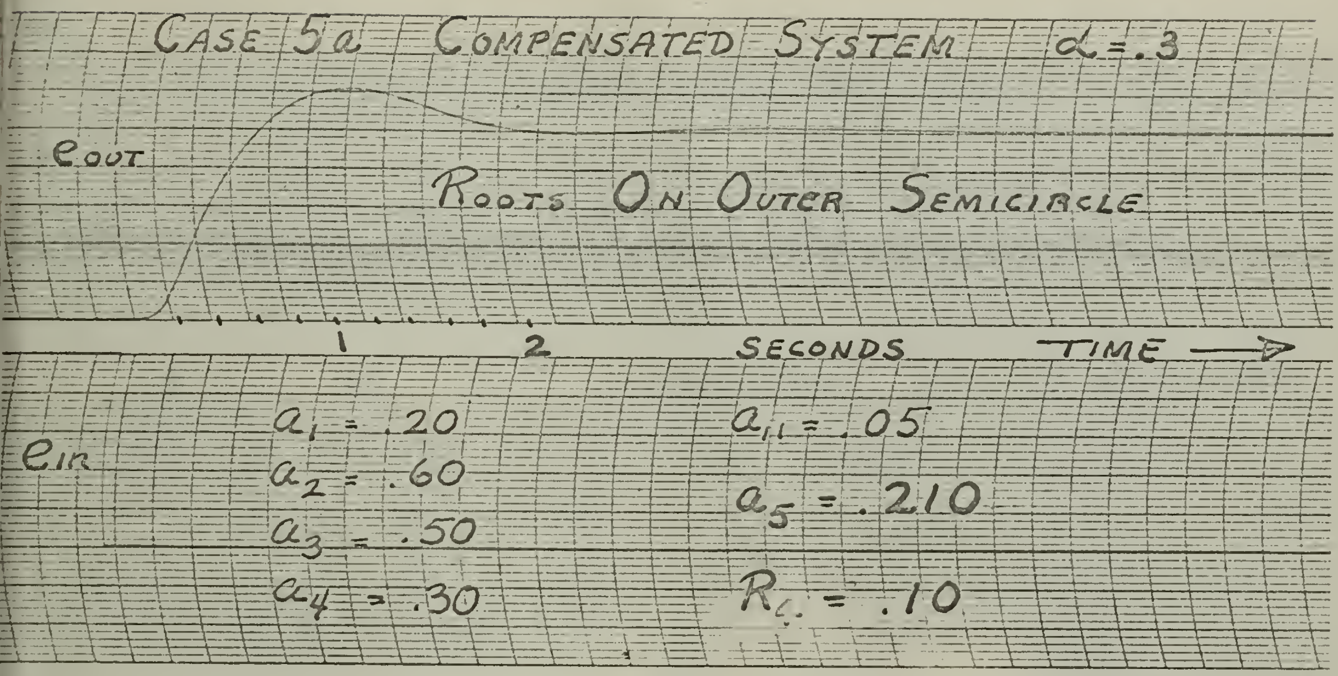
(L)

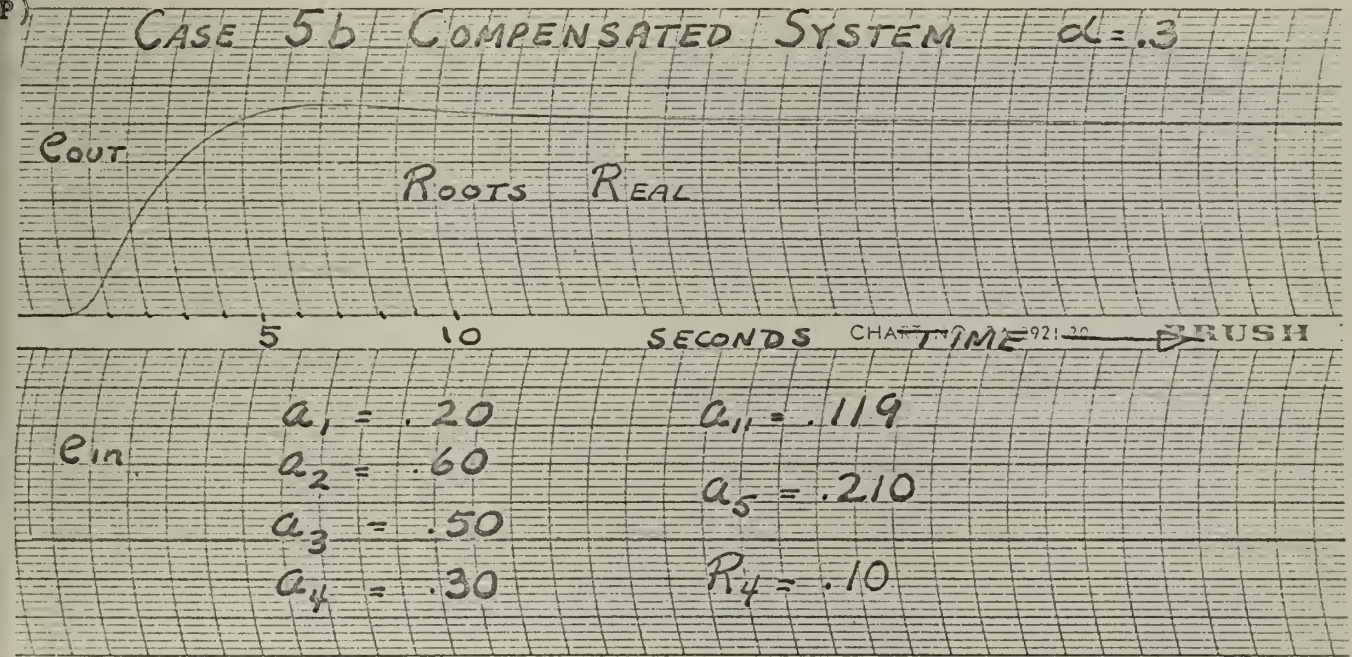


CASE 4a COMPENSATED SYSTEM α = .5
e_{out}
ROOTS ON OUTER SEMICIRCLE
1
2
SECONDS
TIME
CHART NO.
e_{in}
a_1 = .20
a_2 = 1.00
a_3 = .50
a_4 = .50
a_{11} = .052
a_5 = .227
R_4 = .10

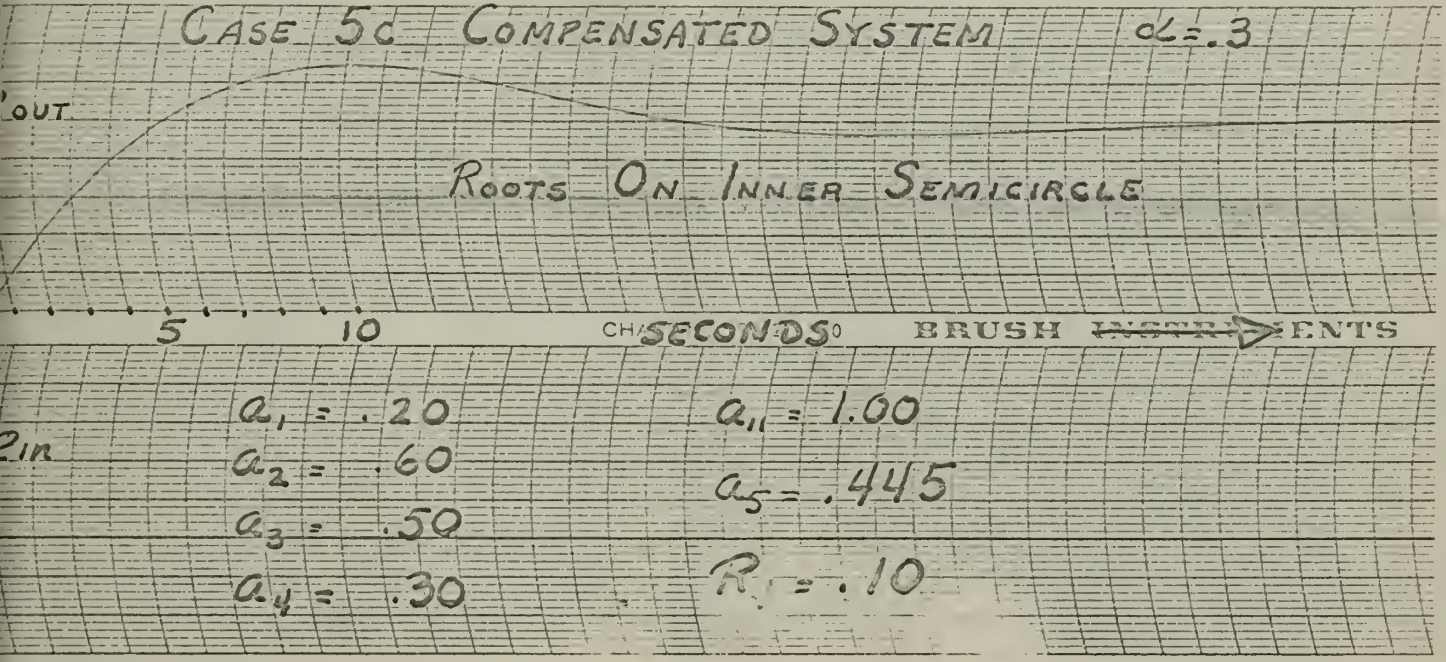

CASE 4b COMPENSATED SYSTEM α = .5
e_{OUT}
ROOTS REAL
5
10
SECONDS
TIME
B
e_{in}
a_1 = .20
a_2 = 1.00
a_3 = .50
a_4 = .50
a_{11} = .106
a_5 = .30
R_4 = .10


CASE 4c COMPENSATED SYSTEM α = .5
e_{OUT}
ROOTS ON INNER SEMICIRCE
10
PRINTED IN U.S.A.
5
10
SECONDS
TIME
e_{in}
a_1 = .20
a_2 = 1.00
a_3 = .50
a_4 = .50
a_{11} = 1.00
a_5 = .227
R_4 = .10

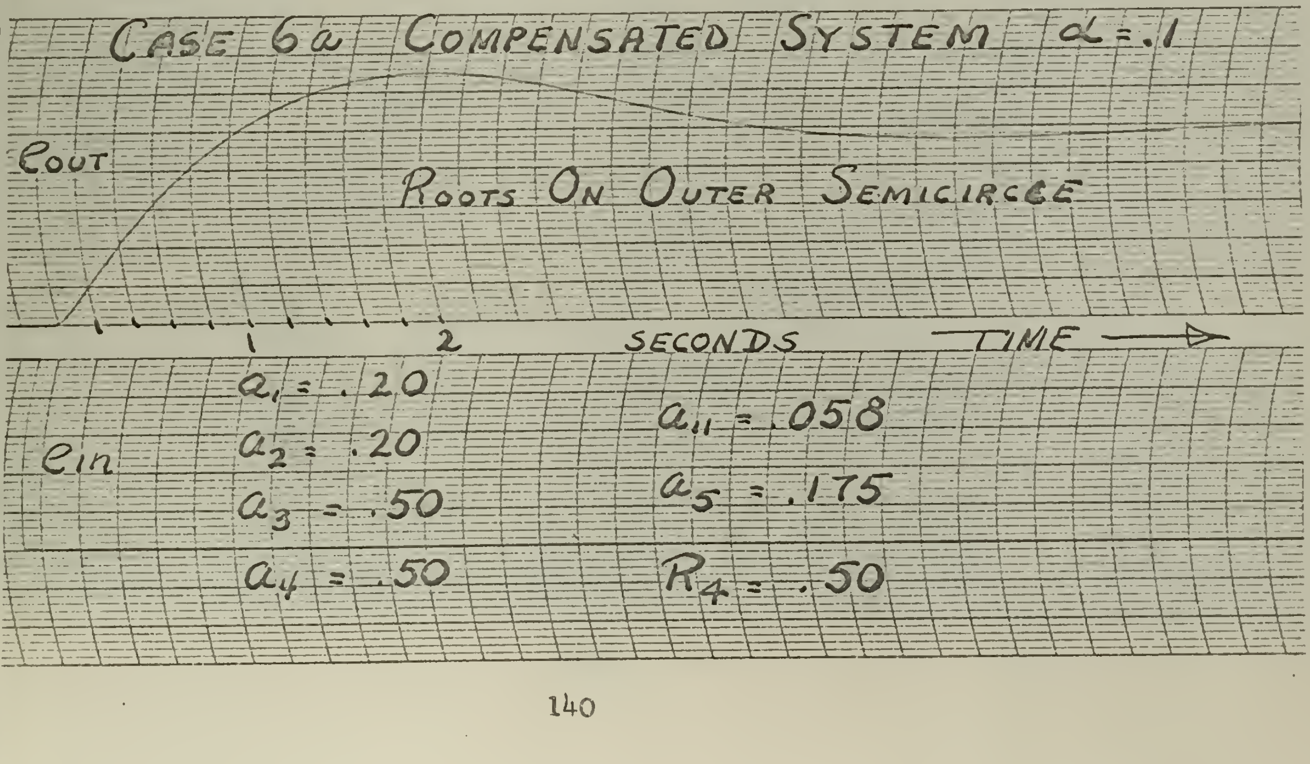

CASE 5a COMPENSATED SYSTEM $\alpha = .3$
e_{OUT}
ROOTS ON OUTER SEMICIRCLE
1
2
SECONDS
TIME
e_{in}
$a_1 = .20$
$a_2 = .60$
$a_3 = .50$
$a_4 = .30$
$a_{11} = .05$
$a_5 = .210$
$R_4 = .10$


CASE 5b COMPENSATED SYSTEM $\alpha = .3$
e_{OUT}
ROOTS REAL
5
10
SECONDS
TIME
CHART
BRUSH
e_{in}
$a_1 = .20$
$a_2 = .60$
$a_3 = .50$
$a_4 = .30$
$a_{11} = .119$
$a_5 = .210$
$R_4 = .10$

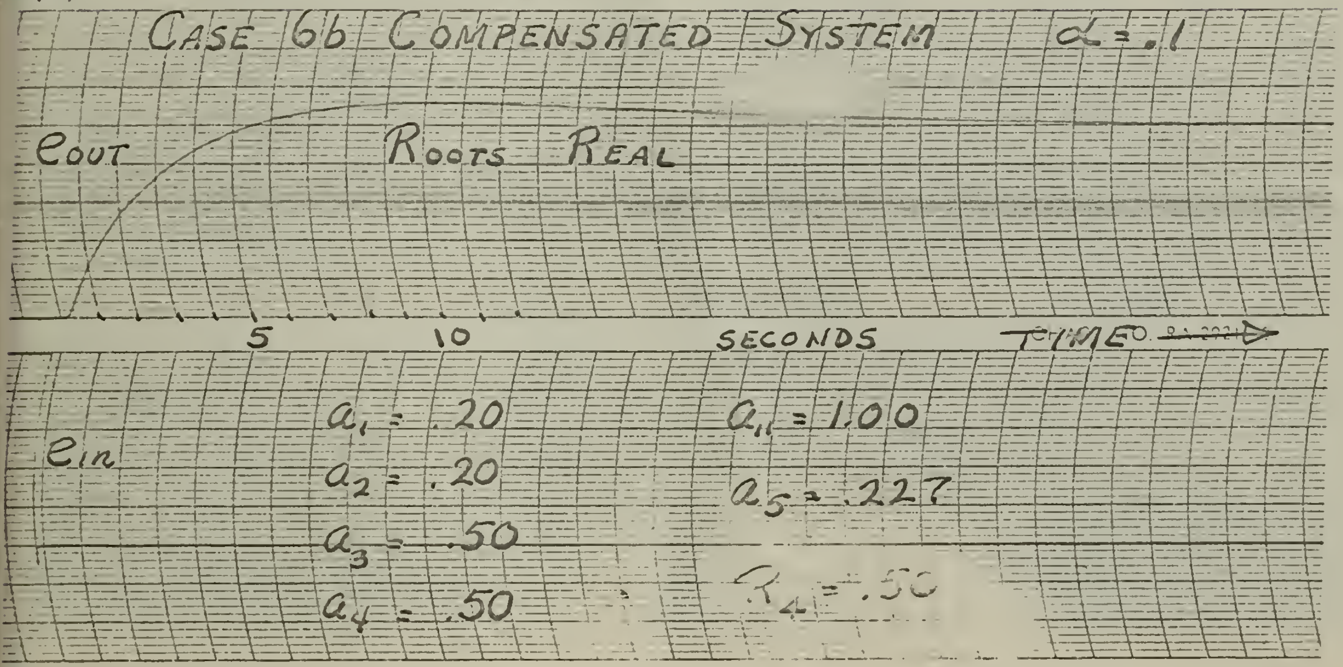
(Q)


CASE 5c COMPENSATED SYSTEM α = .3
eOUT
ROOTS ON INNER SEMICIRCLE
5
10
SECONDS
BRUSH ~~INSTRU~~ENTS
ein
a1 = .20
a2 = .60
a3 = .50
a4 = .30
a11 = 1.00
a5 = .445
R1 = .10

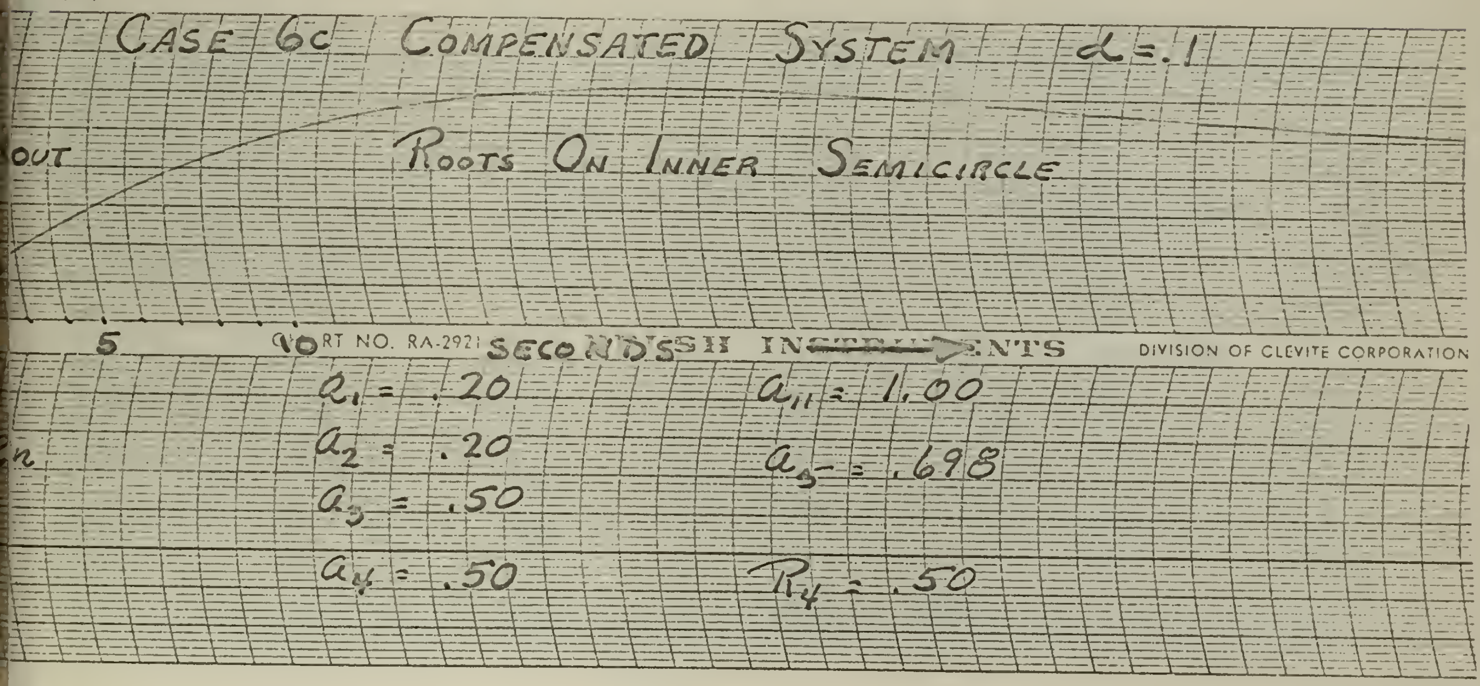
(R)


CASE 6a COMPENSATED SYSTEM α = .1
eOUT
ROOTS ON OUTER SEMICIRCLE
1
2
SECONDS
TIME
ein
a1 = .20
a2 = .20
a3 = .50
a4 = .50
a11 = .058
a5 = .175
R4 = .50

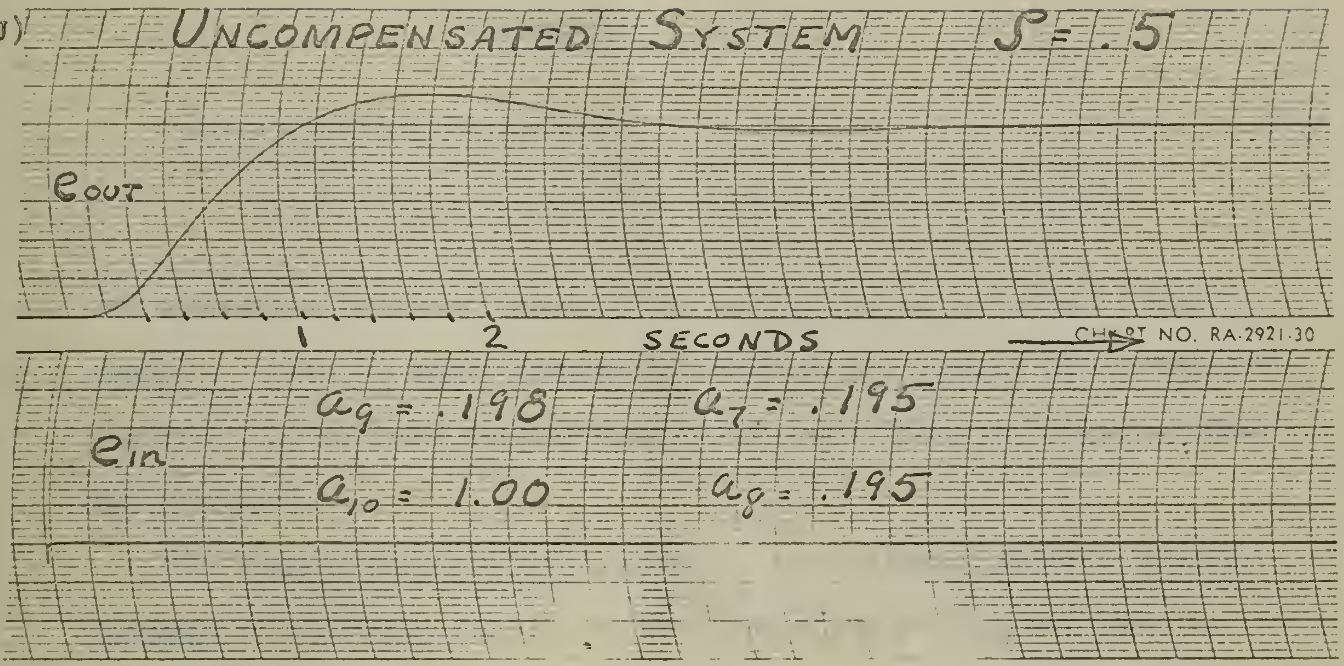
(S)


CASE 6b COMPENSATED SYSTEM α = .1
e_{out}
ROOTS REAL
5
10
SECONDS
TIME
e_{in}
a_1 = .20
a_2 = .20
a_3 = .50
a_4 = .50
a_{11} = 1.00
a_5 = .227
R_4 = .50

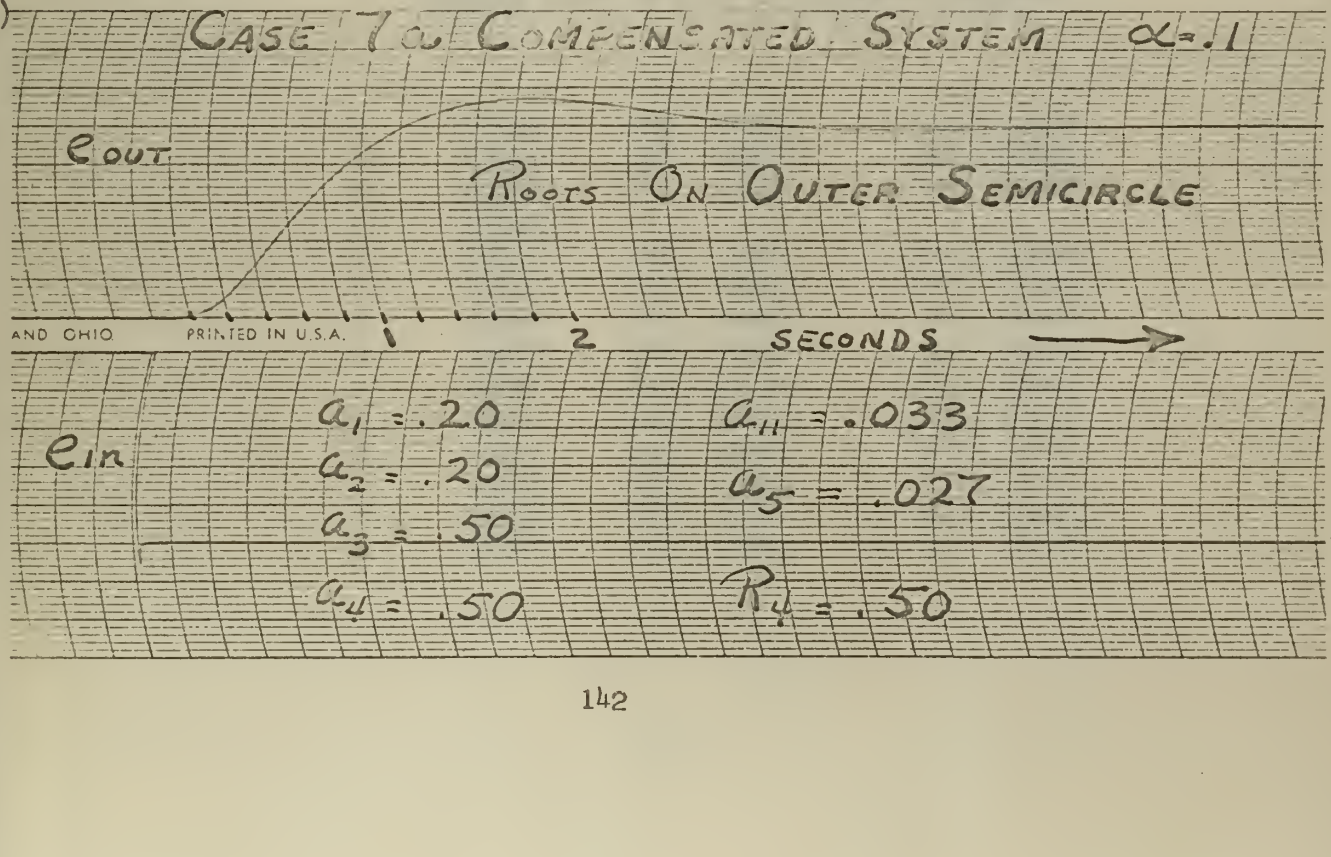
(T)


CASE 6c COMPENSATED SYSTEM α = .1
OUT
ROOTS ON INNER SEMICIRCLE
5
10
SECONDS
RT NO. RA-2921
DIVISION OF CLEVITE CORPORATION
n
a_1 = .20
a_2 = .20
a_3 = .50
a_4 = .50
a_{11} = 1.00
a_5 = .698
R_4 = .50

(U)


UNCOMPENSATED SYSTEM ζ = .5
e_OUT
1
2
SECONDS
CHART NO. RA-2921-30
e_in
a_9 = .198
a_7 = .195
a_10 = 1.00
a_8 = .195

(V)


CASE 7a COMPENSATED SYSTEM α = .1
e_OUT
ROOTS ON OUTER SEMICIRCLE
AND OHIO
PRINTED IN U.S.A.
1
2
SECONDS
e_in
a_1 = .20
a_2 = .20
a_3 = .50
a_4 = .50
a_11 = .033
a_5 = .027
R_4 = .50

APPENDIX C

The characteristic equation

$$s^3 + (K_a K_Z + p + \alpha)s^2 + (K_1 K_Z + p\alpha)s + K_1 K_Z \alpha = 0$$

was solved for different values of p, α, $K_1 K_Z$, etc., using a digital computer. The 1604 Model Computer produced by the Control Data Corporation was used. It provided answers rapidly and easily once the proper program was developed.

The program itself which is given on the following pages in machine language is utilizing the Lin-Bairstow method for solving the roots of a polynomial equation.

PROGRAM USED ON 1604 IN MACHINE LANGUAGE

```
0       63000     200  63003        500  00000
0       63001     754  63005        500  00000
0       63002     760  63002        500  00000
0       63003
0       63004     760  63004        500  00000
0       63005     750  00000        565  63142
0       63006     505  00000        100  00000
0       63007     205  63675        500  00000
0       63010     545  00131        750  63007
0       63011     120  63177        200  63410
0       63012     200  63542        561  63140
0       63013     572  63140        563  63141
0       63014     574  63141        523  63005
0       63015     123  00001        513  00002
```

```
0     63016     563  63005     600  63045
0     63017     600  63143     610  63147
0     63020     610  63076     600  63077
0     63021     600  63120     110  00001
0     63022     600  63122     610  63102
0     63023     610  63143     600  63150
0     63024     600  63103     030  00030
0     63025     600  63046     610  63145
0     63026     600  63034     610  63034
0     63027     600  63035     110  00001
0     63030     600  63050     110  00001
0     63031     610  63053     050  00017
0     63032     070  00011     600  63037
0     63033     600  63036     504  00000
0     63034     124  00000     330  00000
0     63035     204  00000     500  00000
0     63036     544  00000     750  63034
0     63037     100  00000     150  63174
0     63040     600  63037     600  63055
0     63041     610  63063     610  63130
0     63042     600  63146     500  00000
0     63042     110  77776     600  63064
0     63044     501  00000     503  00001
0     63045     133  00000     203  63202
0     63046     320  00000     203  63256
0     63047     203  63254     553  63045
0     63050     300  00000     201  63411
0     63051     320  63203     201  63260
0     63052     121  63411     320  63202
0     63053     301  63257     301  00000
0     63054     201  63412     500  00000
0     63055     541  00000     750  63051
0     63056     120  63256     300  63411
0     63057     200  63543     500  00000
0     63060     320  63203     201  63256
0     63061     120  63202     321  63543
0     63062     301  63255     301  63412
0     63063     201  63544     502  00000
0     63064     541  00000     750  63060
0     63065     312  63411     512  77776
0     63066     332  63543     200  63210
0     63067     512  77776     322  63543
0     63070     512  00001     312  63543
0     63071     200  63211     512  77776
0     63072     122  63543     512  00001
0     63073     332  63543     512  00002
0     63074     322  63411     512  77776
0     63075     312  63411     330  63211
0     63076     200  63206     300  00000
0     63077     200  00000     122  63411
0     63100     320  63210     512  00001
0     63101     312  63411     330  63211
0     63102     200  63207     300  00000
```

```
0      63103      200  00000        500  00000
0      63104      360  63206        750  63106
0      63105      130  63206        200  63206
0      63106      360  63207        750  63110
0      63107      130  63207        200  63207
0      63110      120  63201        501  00002
0      63111      651  63206        750  63117
0      63112      545  00063        750  63044
0      63113      120  63116        700  63201
0      63114      650  63212        750  63002
0      63115      750  63044        500  00000
0      63116      000  30000        000  00000
0      63117      120  63213        200  63201
0      63120      130  00000        330  63200
0      63121      200  63204        320  63204
0      63122      310  00000        223  63155
0      63123      040  00000        220  63126
0      63124      754  63216        500  00000
0      63125      500  00000        500  00000
0      63126      070  00060        120  63204
0      63127      204  63675        214  63677
0      63130      514  00003        100  00000
0      63131      501  00003        500  00000
0      63132      641  63174        750  63145
0      63133      121  63174        220  63140
0      63134      110  77776        220  63136
0      63135      750  63160        500  00000
0      63136      130  63411        204  63677
0      63137      120  63177        204  63676
0      63140      501  00000        502  00000
0      63141      503  00000        504  00000
0      63142      505  00000        120  63200
0      63143      200  63214        200  63215
0      63144      750  63005        500  00000
0      63145      123  63410        203  00000
0      63146      543  00000        750  63145
0      63147      120  63200        200  63214
0      63150      200  63215        500  00000
0      63151      120  63076        600  63154
0      63152      110  00001        610  63153
0      63153      100  00000        200  00000
0      63154      200  00000        750  63037
0      63155      200  63205        130  63205
0      63156      170  63177        214  63676
0      63157      040  00000        750  63124
0      63160      130  63411        330  63410
0      63161      330  63200        200  63171
0      63162      320  63171        200  63172
0      63163      130  63412        330  63410
0      63164      300  63172        200  63173
0      63165      100  77775        700  63130
0      63166      120  63171        200  63204
0      63167      120  63173        223  63155
```

```
0        63170         750  63124         500  00000
0        63171
0        63172         200  14431         745  33734
0        63173         577  61607         713  14407
0        63174         000  00000         000  00002
0        63175         000  00000         000  00001
0        63176
0        63177         200  14000         000  00000
0        63200         200  24000         000  00000
0        63201         175  04000         000  00000
0        63202         200  24455         406  25126
0        63203
0        63204
0        63205         577  61660         317  52411
0        63206
0        63207         173  67401         377  30510
0        63210
0        63211
0        63212         176  64000         000  00000
0        63213         175  04000         000  00000
0        63214         200  24000         000  00000
0        63215         200  24000         000  00000
0        63216         750  00000         561  63241
0        63217         220  63237         223  63216
0        63220         500  00000         420  63245
0        63221         050  00001         030  00045
0        63222         223  63223         110  00002
0        63223         110  03777         030  00001
0        63224         050  00044         200  63250
0        63225         440  63246         010  00004
0        63226         233  63227         010  00001
0        63227         200  63251         140  63244
0        63230         200  63252         120  63243
0        63231         270  63252         140  63242
0        63232         501  00001         040  00000
0        63233         200  63252         120  63251
0        63234         270  63252         140  63252
0        63235         010  00001         551  63233
0        63236         010  00011         420  63250
0        63237         320  63247         521  63216
0        63240         511  00001         571  63241
0        63241         501  00000         750  00000
0        63242         213  66405         014  47436
0        63243         763  72106         436  47567
0        63244         030  56173         527  71341
0        63245         200  00000         000  00000
0        63246         377  77777         777  74000
0        63247         200  14000         000  00000
0
0   *    63400
S4           STA    0 00004
```

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